Applications of the spectral/*hp* element method to complex flow geometries

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Overview

- Motivation
- Challenges of transient flow simulations
- Results
- Future work

Motivation

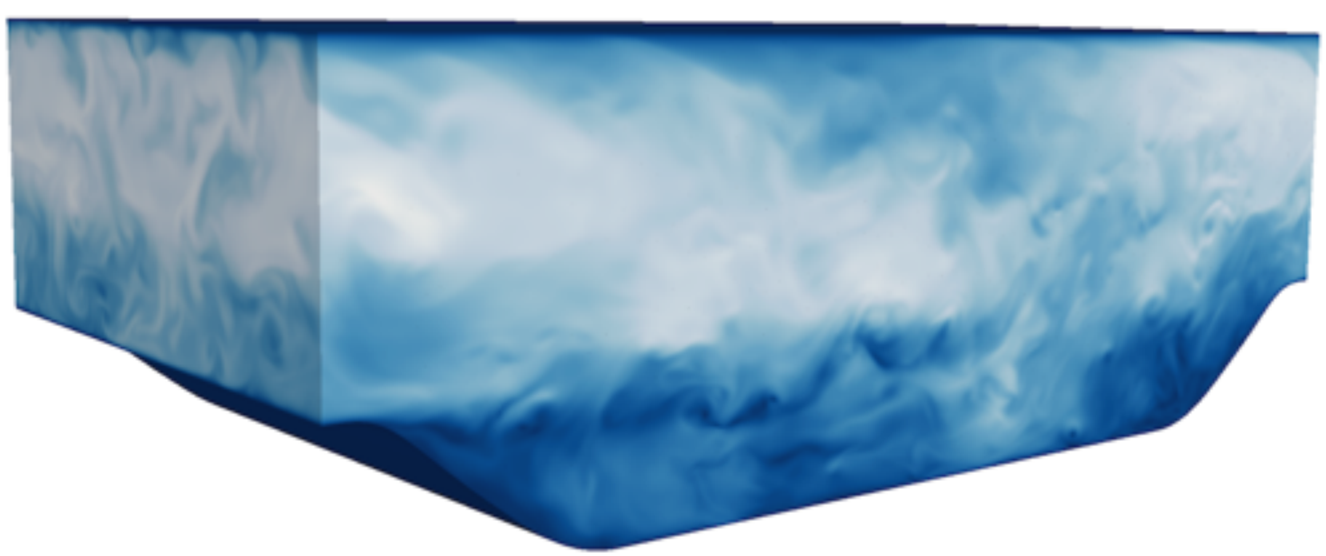
Primary research goal is to investigate challenging external aerodynamics cases:

- High Reynolds numbers
- Complex three-dimensional geometries
- Large resolution requirements
- Transient dynamics

Motivation

- (Fully resolved) DNS gives extremely accurate results but is too expensive for high-Re simulations.

- How can we apply existing efficient academic DNS codes for industrial applications?

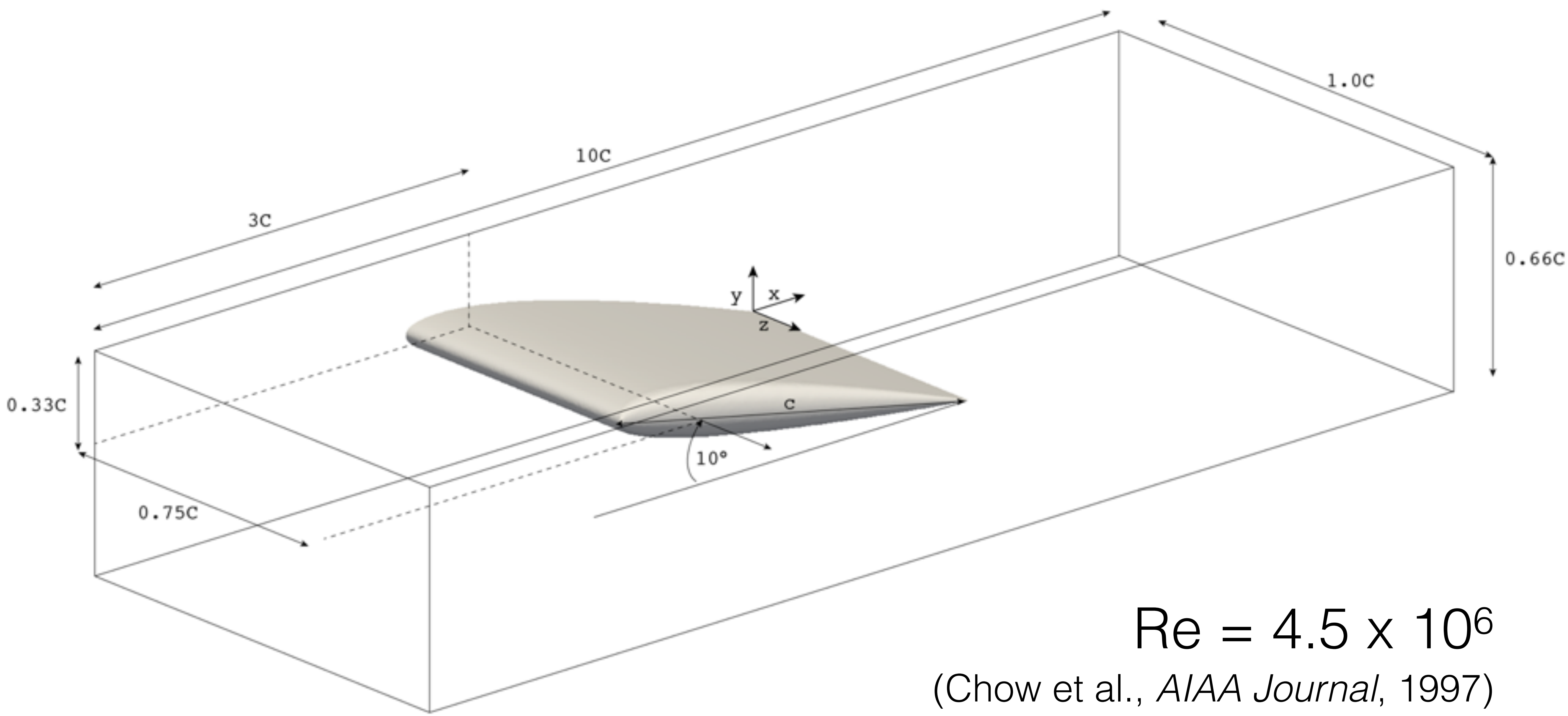


DNS of periodic hill

2D spectral element + 1D Fourier spectral

~25 million dof

NACA 0012 wing tip

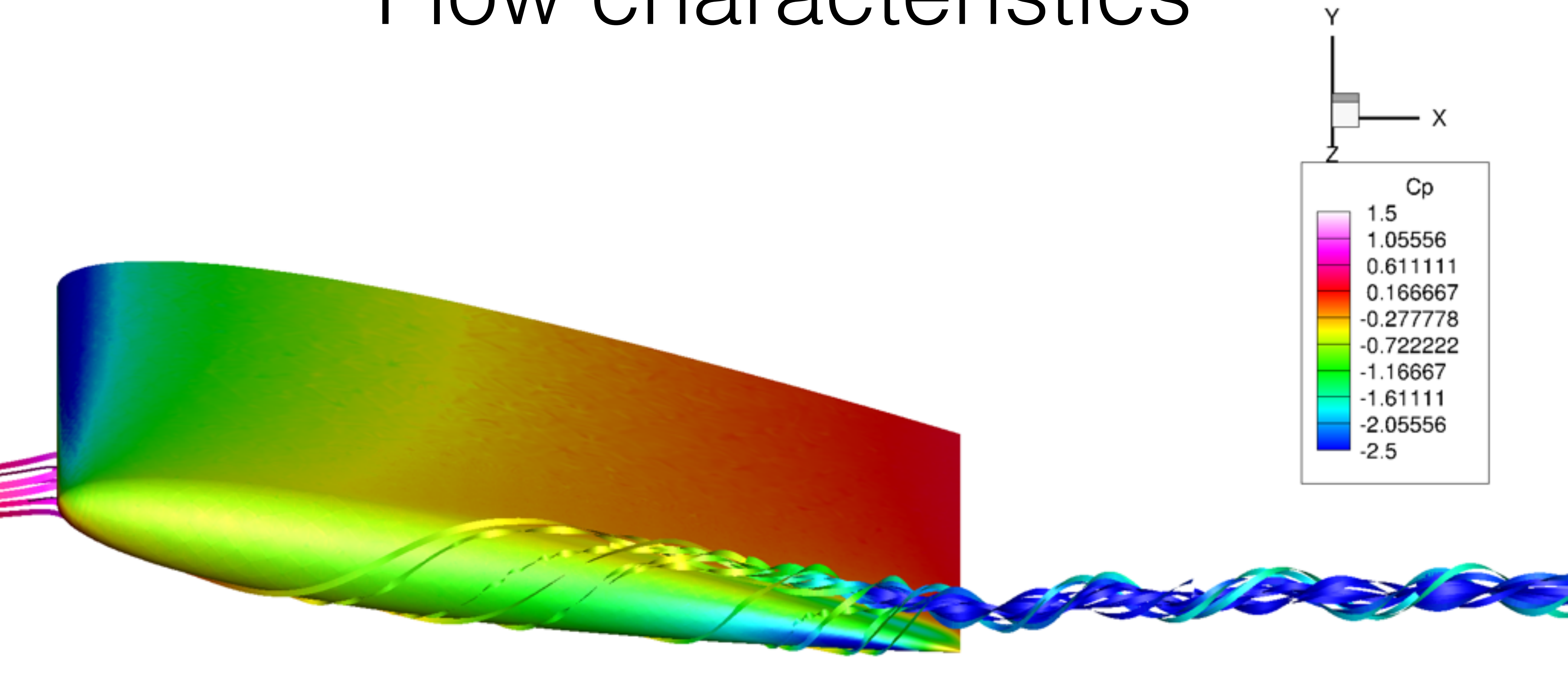


Re = 4.5×10^6

(Chow et al., *AIAA Journal*, 1997)

Difficult to capture transient effects with RANS

Flow characteristics



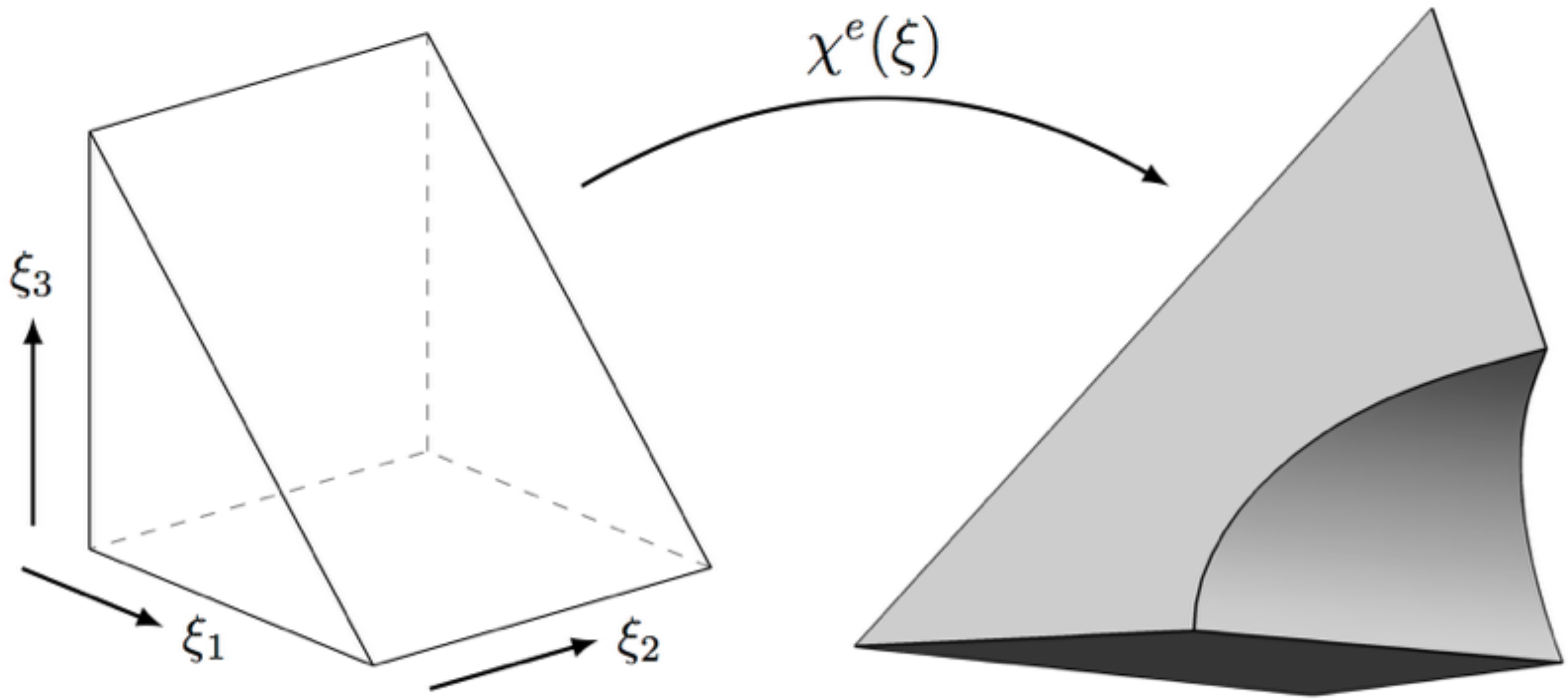
How low can we go in terms of resolution and still capture essential flow features?

High-order mesh generation

Boundary layer grids are hard to generate:

- High shear near walls
- First element needs to be of size roughly $O(Re^{-2})$
- Unfeasible to run with this number of elements in the entire domain and across surface of wall
- Therefore highly-stretched elements required
- Also has to be coarse for high-order to make sense

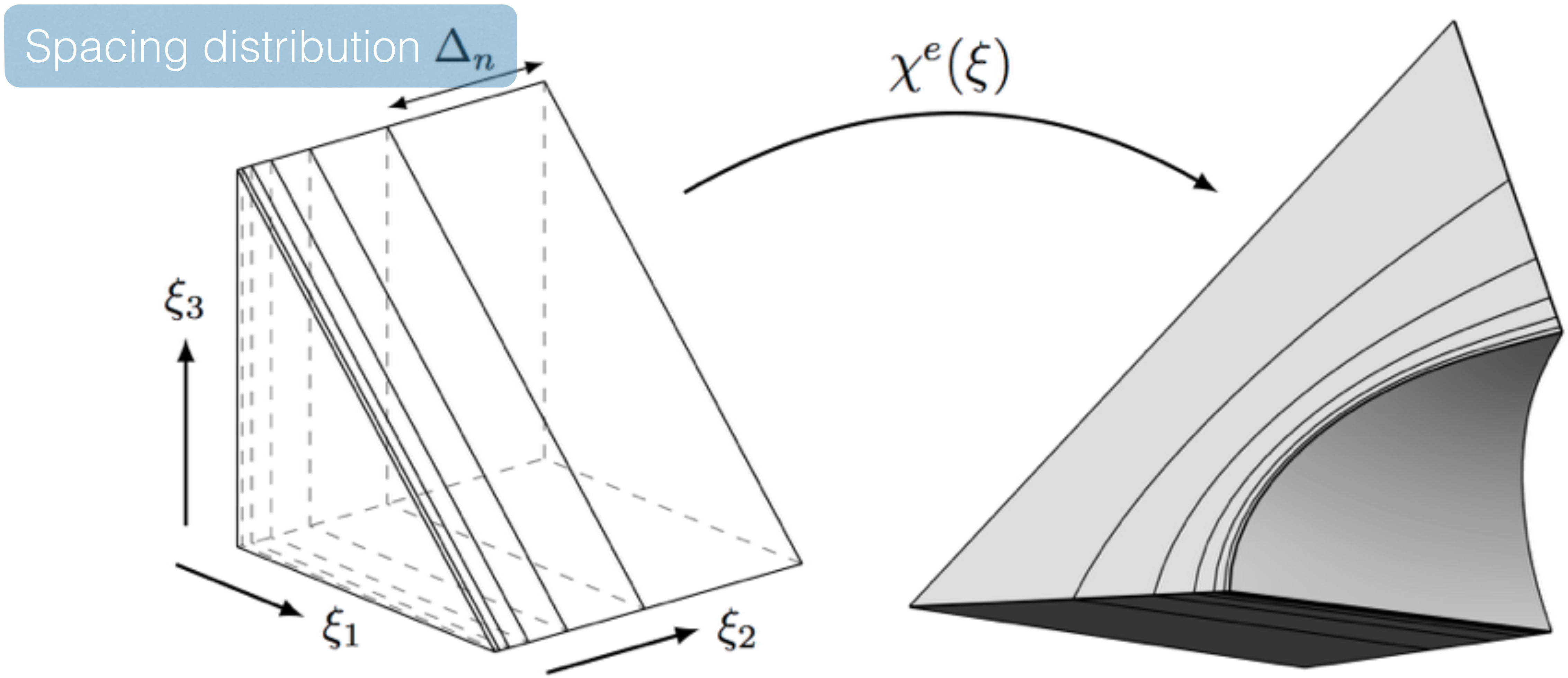
Isoparametric mapping



Shape function is a mapping from
reference element (parametric coordinates) to
mesh element (physical coordinates)

An isoparametric approach to high-order curvilinear boundary-layer meshing
D. Moxey, M. Hazan, S. J. Sherwin, J. Peiró, Comp. Meth. Appl. Mech. Eng. **283**, 636-650 (2015).

Boundary layer mesh generation



Subdivide the reference element in order to obtain a boundary layer mesh

An isoparametric approach to high-order curvilinear boundary-layer meshing
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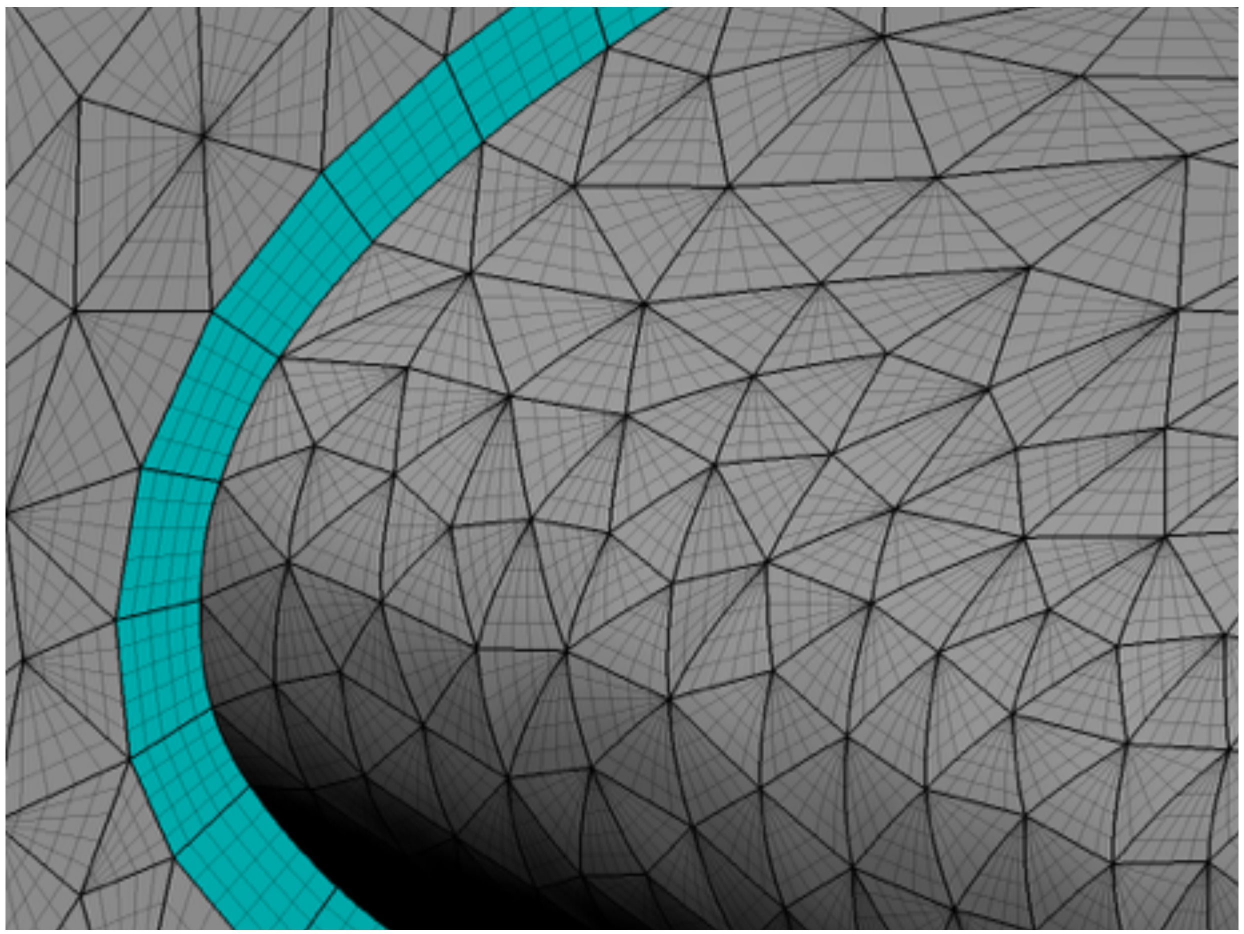
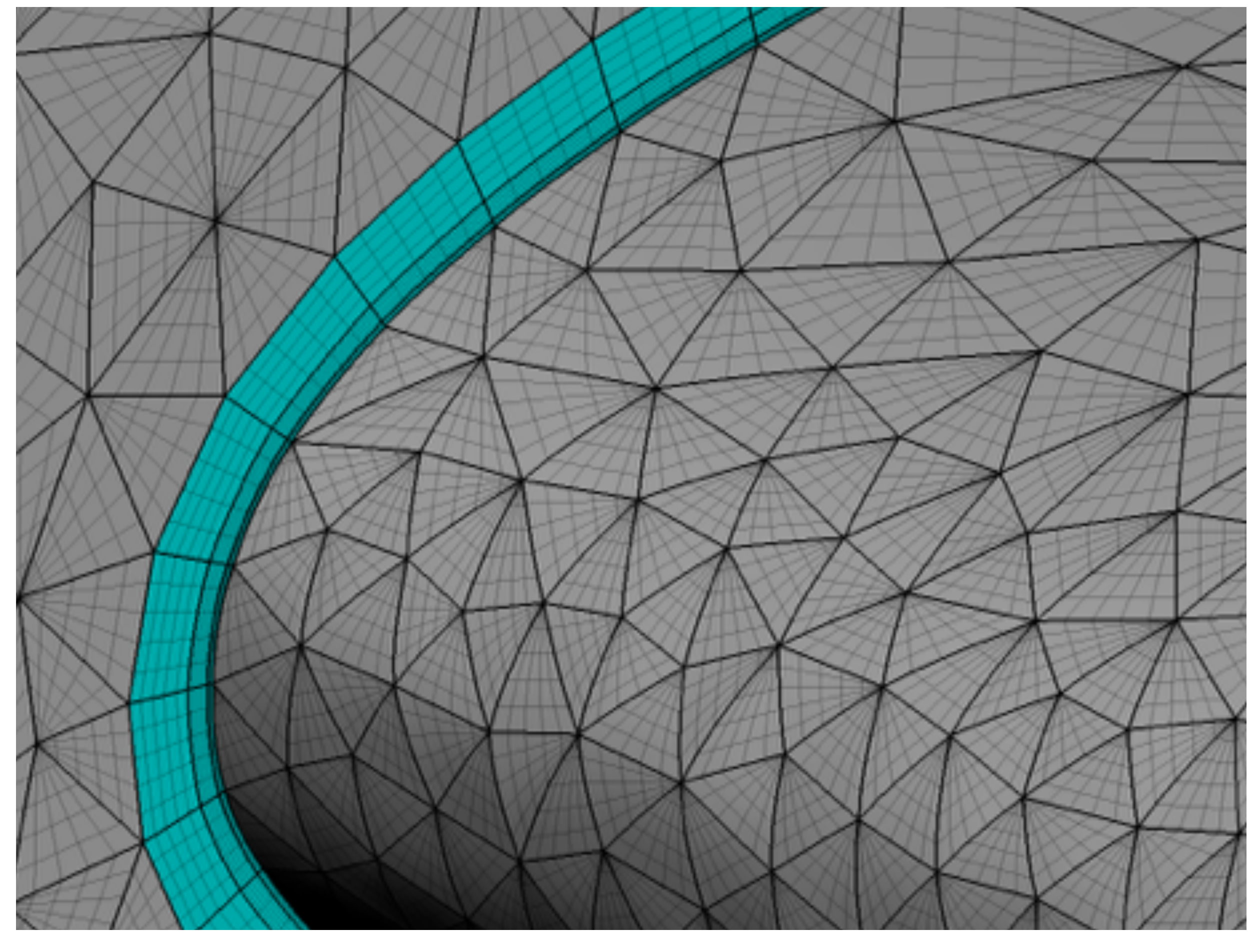
NACA 0012 boundary layer grid

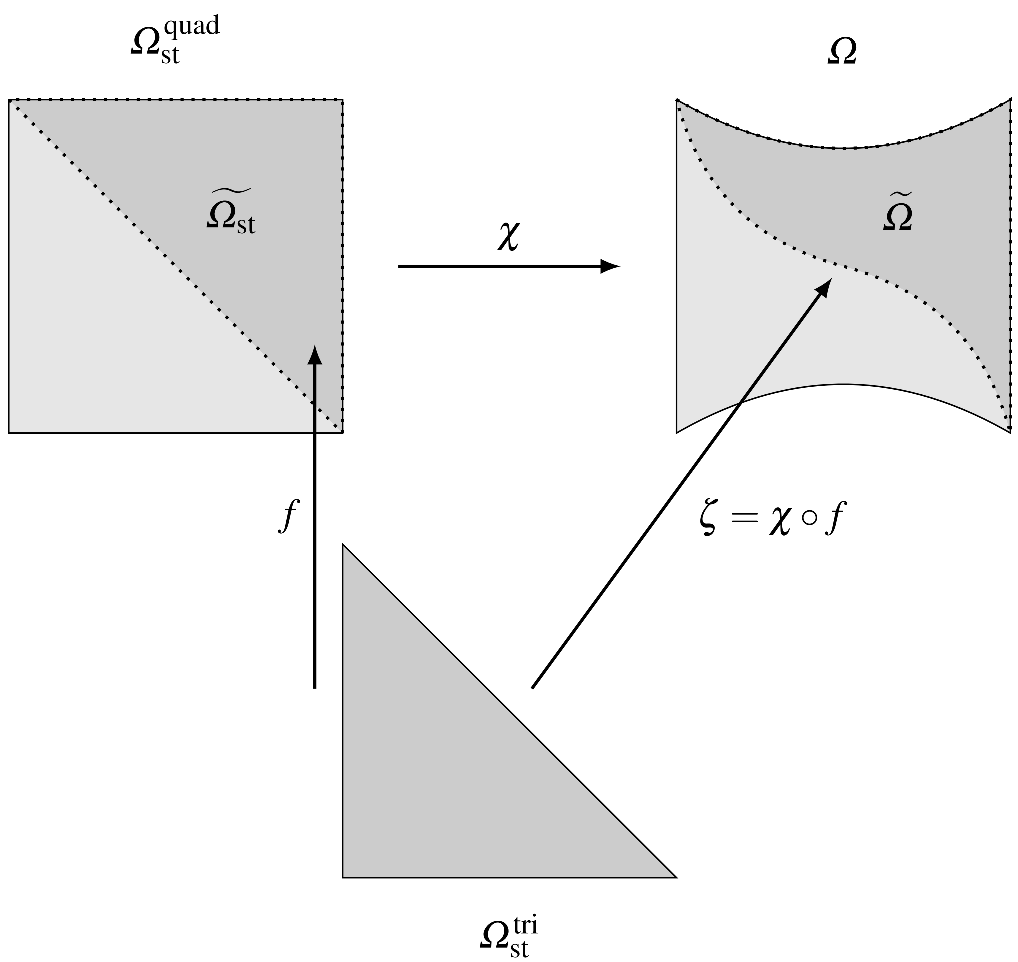


High order mesh
$P = 5$

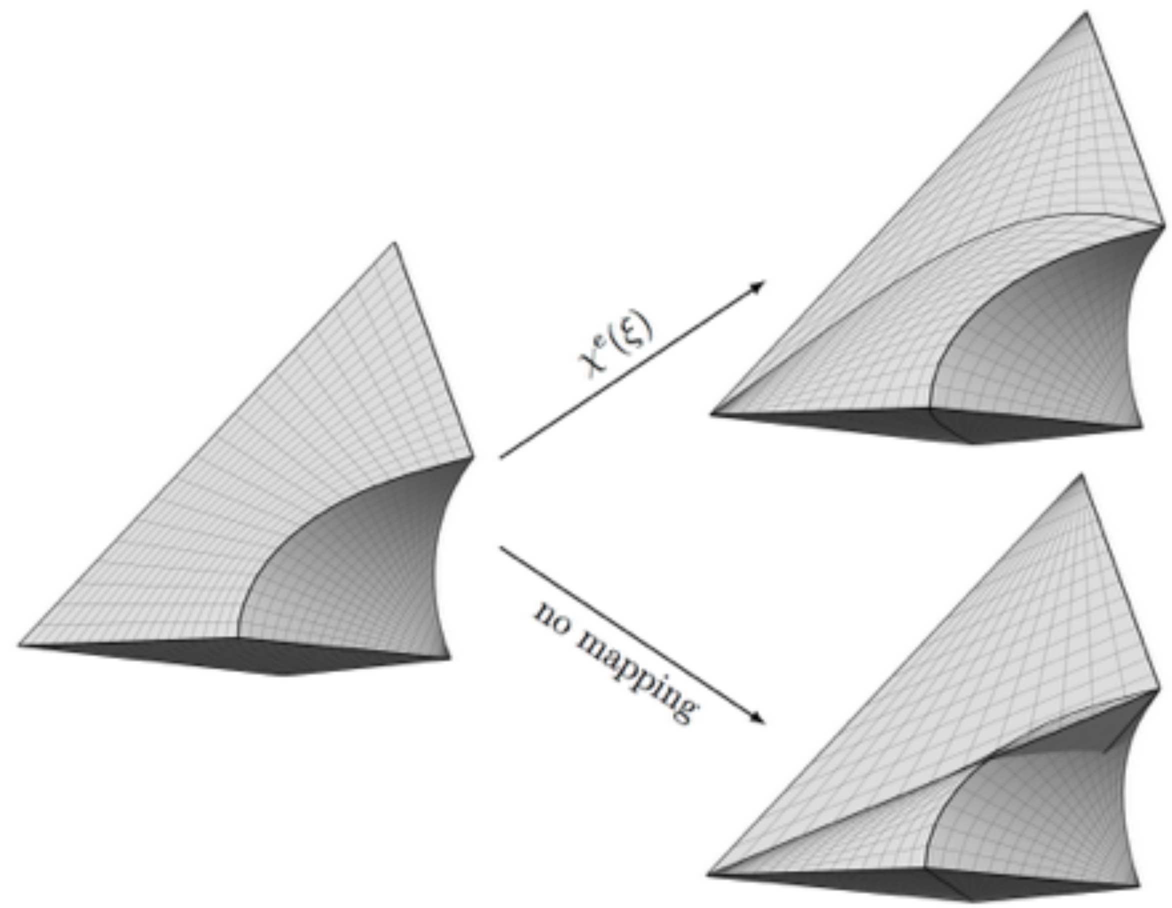


After splitting

More complex transforms



Quads to triangles



Prisms to tetrahedra

On the generation of curvilinear meshes through subdivision of isoparametric elements
D. Moxey, M. Hazan, S. J. Sherwin, J. Peiró, to appear in proceedings of Tetrahedron IV

Stabilisation

Very high Reynolds numbers + under-resolution will inevitability cause instability. Common causes:

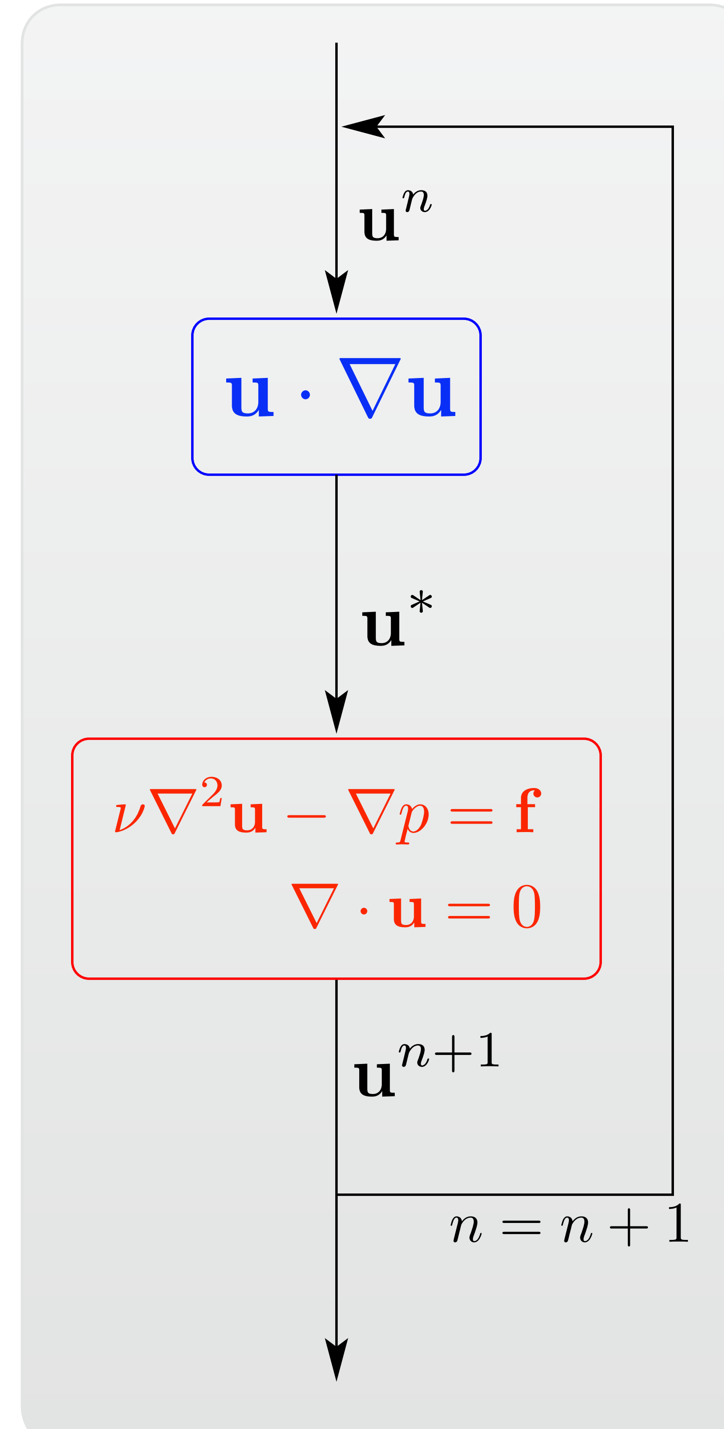
- Consistent integration of nonlinear terms
- Insufficient dissipation from the numerical method

Here we use

- Consistent integration of nonlinear terms
- Spectral vanishing viscosity

Navier-Stokes Solver

Navier–Stokes:
$$\partial_t \mathbf{u} + \mathbf{N}(\mathbf{u}) = -\nabla p + \nu \nabla^2 \mathbf{u}$$
$$\nabla \cdot \mathbf{u} = 0$$



Velocity correction scheme (aka stiffly stable):

Orszag, Israeli, Deville (90), Karnaidakis Israeli, Orszag (1991), Guermond & Shen (2003)

Advection: $\mathbf{u}^\star = -\sum_{q=1}^{J} \alpha_q \mathbf{u}^{n-q} - \Delta t \sum_{q=0}^{J-1} \beta_q \mathbf{N}(\mathbf{u}^{n-q})$

Pressure Poisson: $$\nabla^2 p^{n+1} = \frac{1}{\Delta t} \nabla \cdot \mathbf{u}^*$$

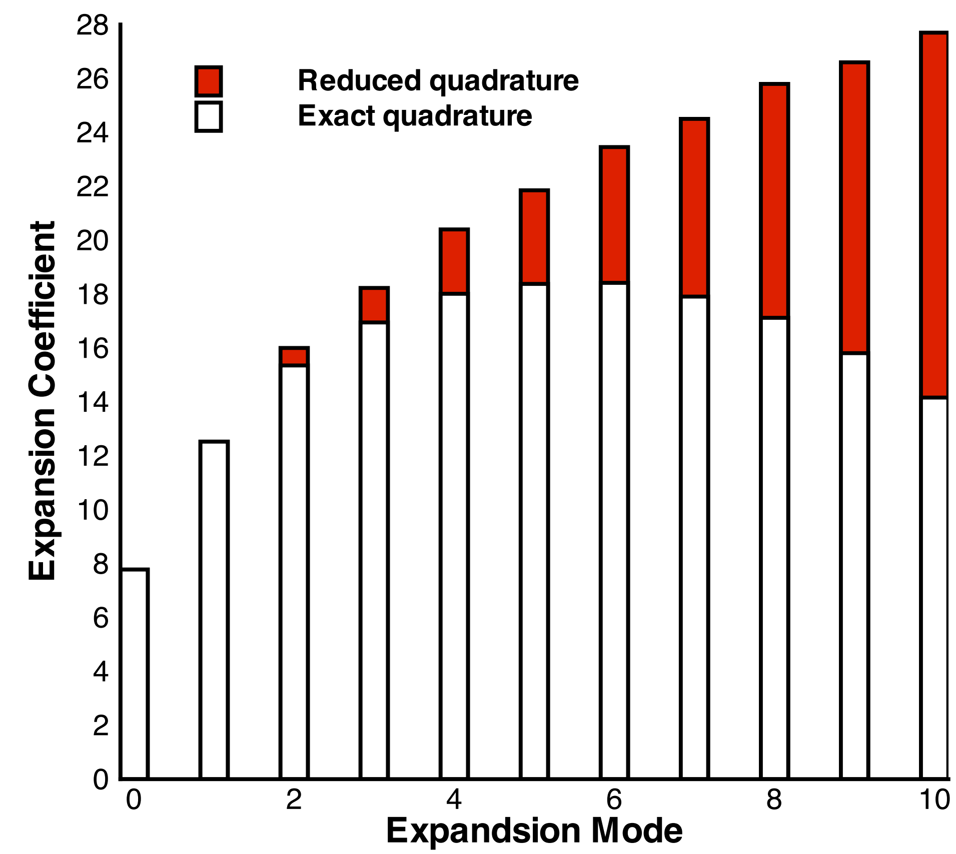
Helmholtz: $\nabla^2 \mathbf{u}^{n+1} - \frac{\alpha_0}{\nu \Delta t} \mathbf{u}^{n+1} = -\frac{\mathbf{u}^\star}{\nabla \Delta t} + \frac{1}{\nu} \nabla p^{n+1}$

Aliasing

Example: $u(\xi) = \sum_{k=0}^{10} u_k \psi_i(\xi)$

Galerkin projection of u^2 using:

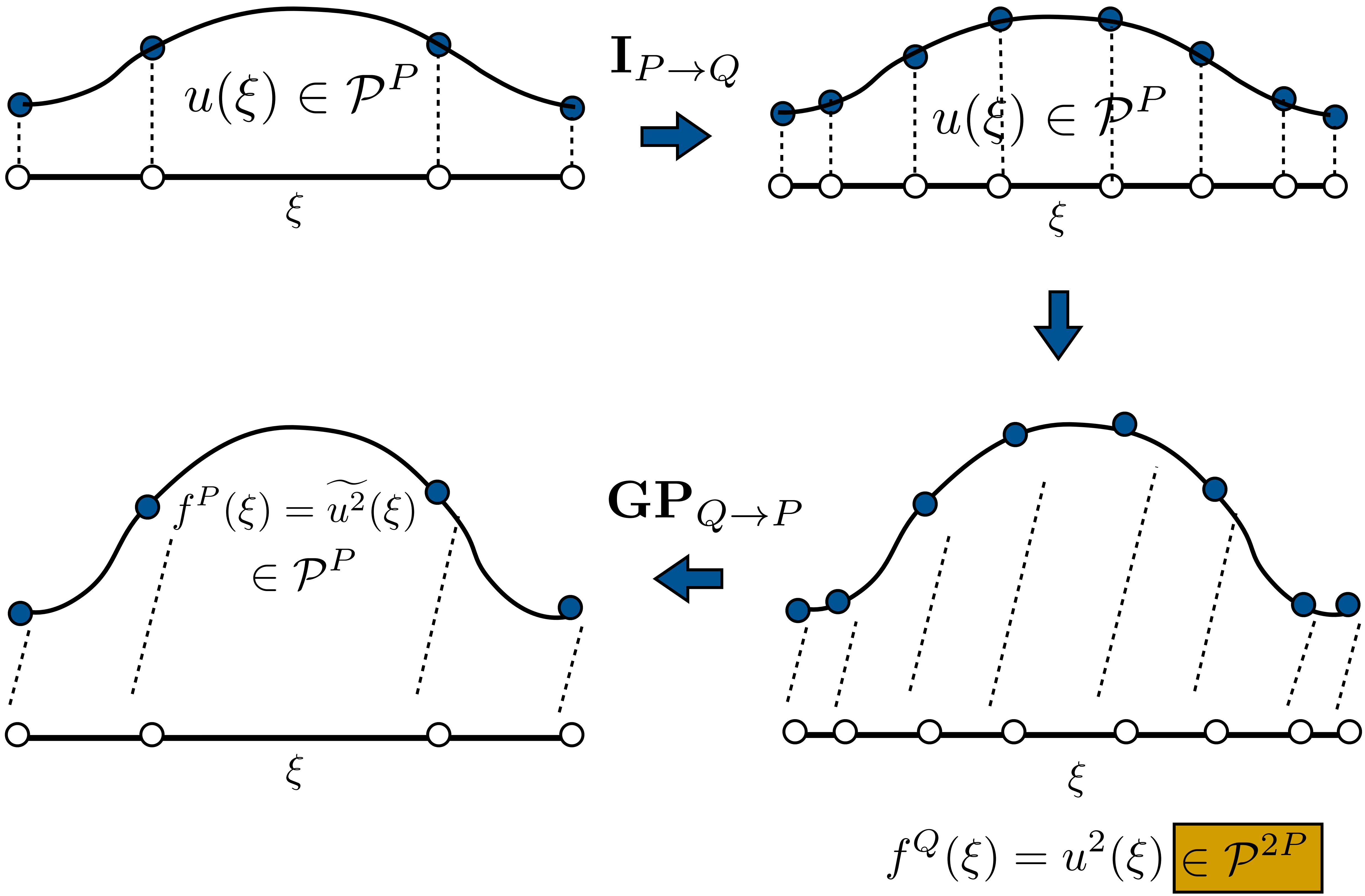
- Q = 17 – exact Quadrature
- Q = 12 – sufficient for integrating 20th degree polynomials



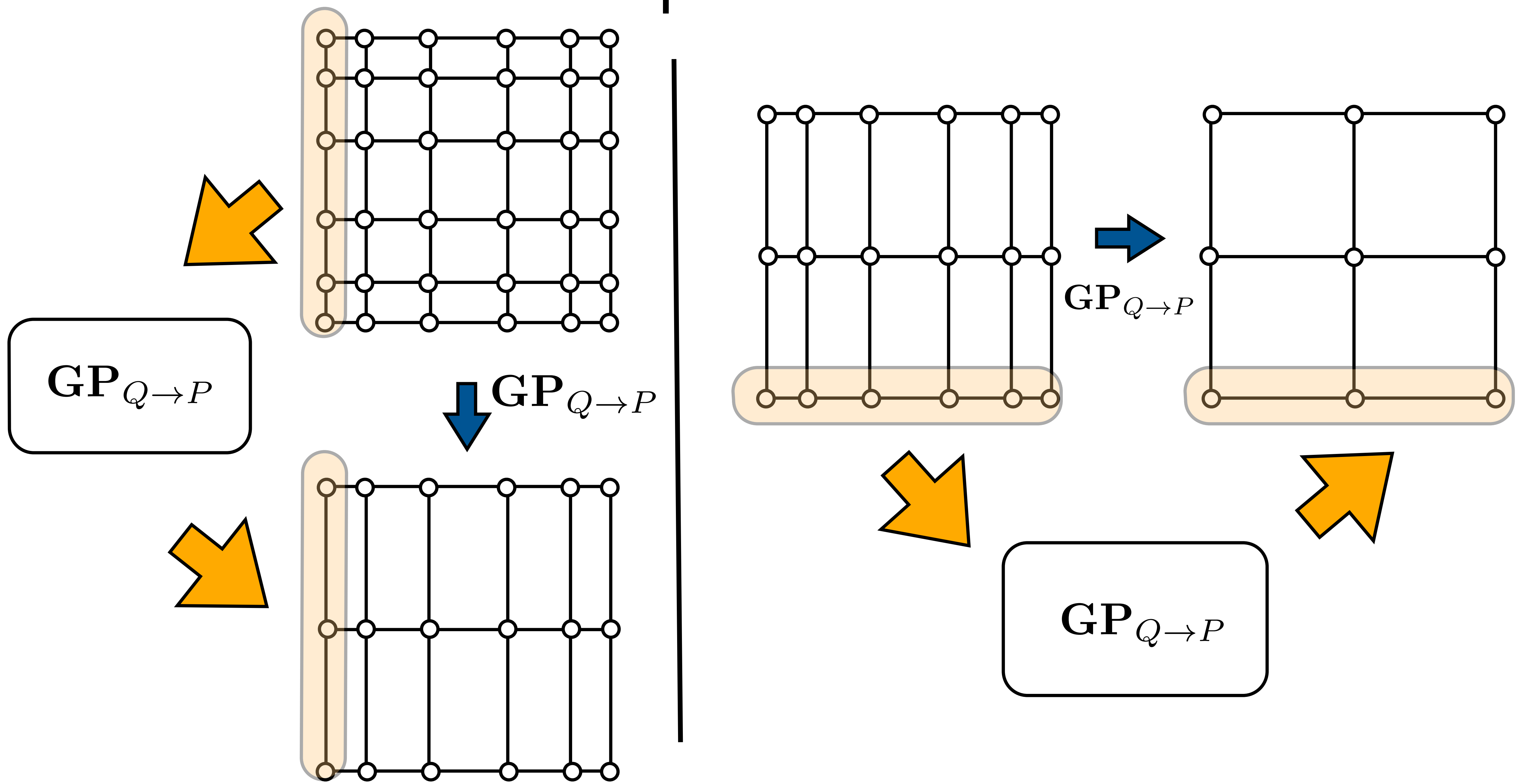
Aliasing Error

Example from Kirby & Karniadakis, J. Comp. Phys (2003)

Overview of nodal projection of u^2



Use tensor product structure



$$P \times Q^2 + P^2 \times Q \Rightarrow O(P^3)$$

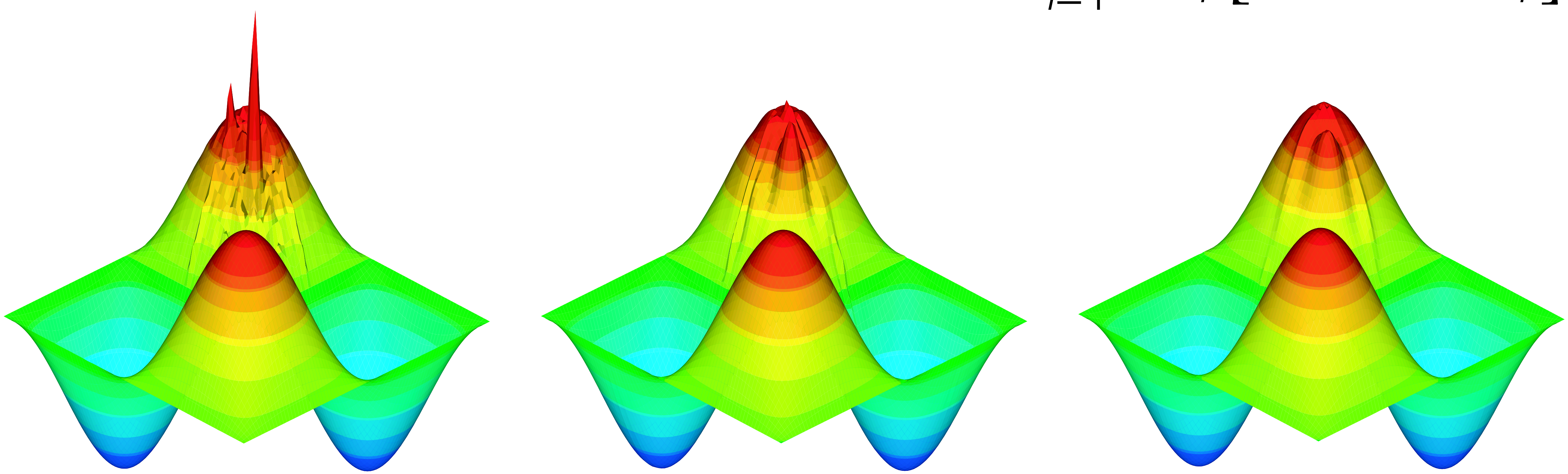
Essentially performing sum factorisation

In 3D: $O(P^6)$ vs. $O(P^4)$

Dealiasing techniques for high-order spectral element methods on regular and irregular grids
G. Mengaldo, D. de Grazia, D. Moxey, P. Vincent, S. J. Sherwin, under review in *J. Comp. Phys.*

Spectral Vanishing Viscosity

$$\frac{\partial u}{\partial t} = v\nabla^2 u + S_{VV}(u), \qquad S_{VV}(u) = \varepsilon \sum_{i=1}^{\dim} \frac{\partial}{\partial x_i}\left[Q_{\dim} \star \frac{\partial u}{\partial x_i}\right]$$



No SVV

$P_{cut} = 7,$

$\epsilon_{SVV} = 0.1$

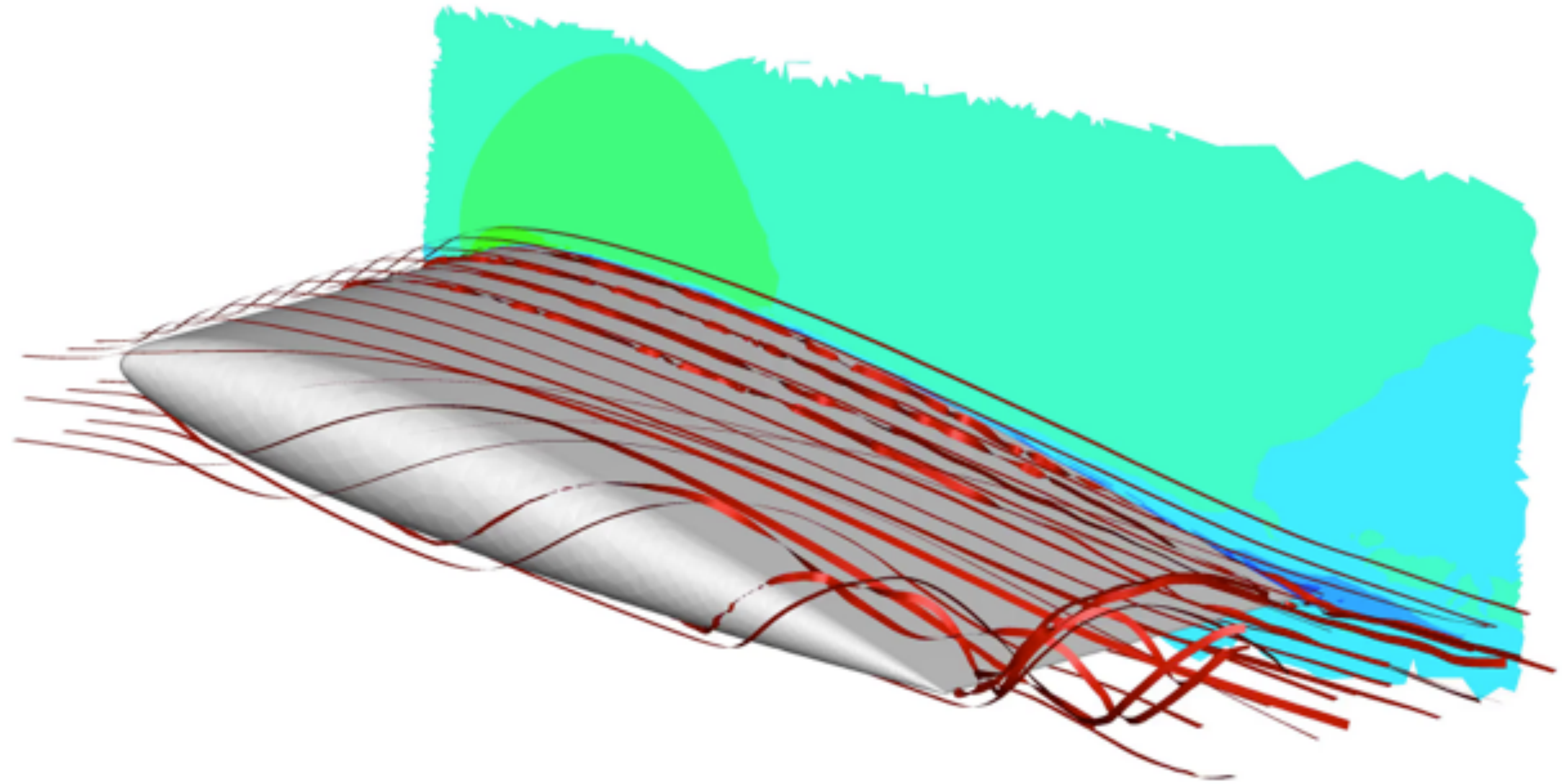
$P_{cut} = 3,$

$\epsilon_{SVV} = 0.1$

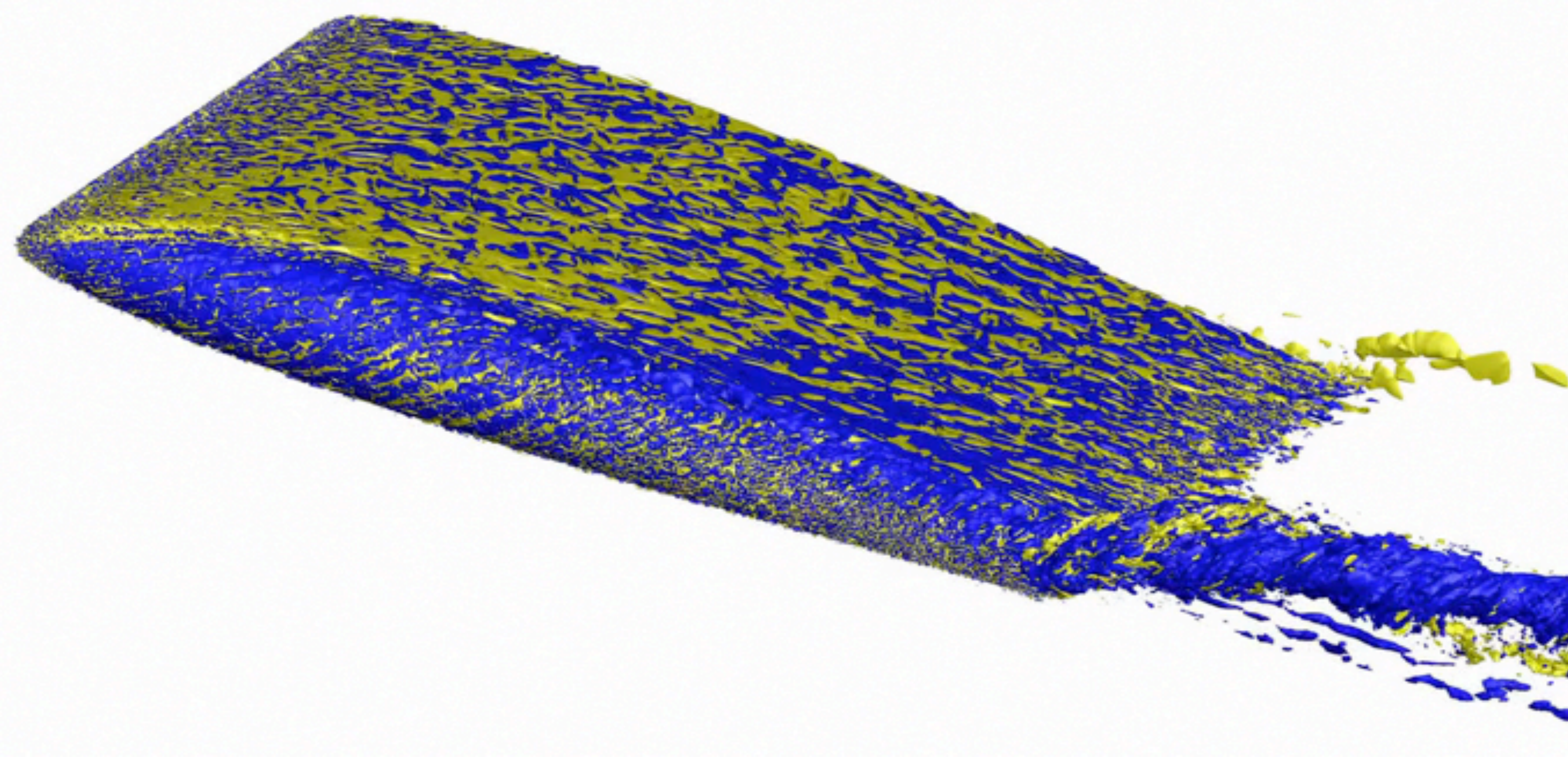
R. Kirby, S. Sherwin, *Comp. Meth. Appl. Mech. Eng.*, 2006

NACA 0012 wing tip (Re = 1.2M)

Streamlines



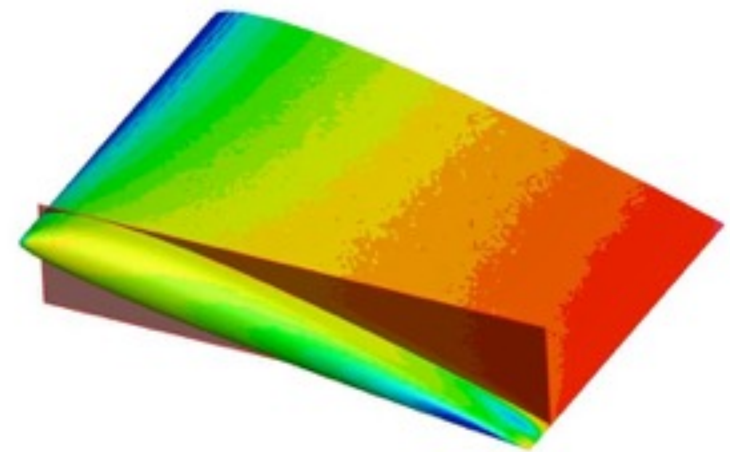
Streamwise vorticity



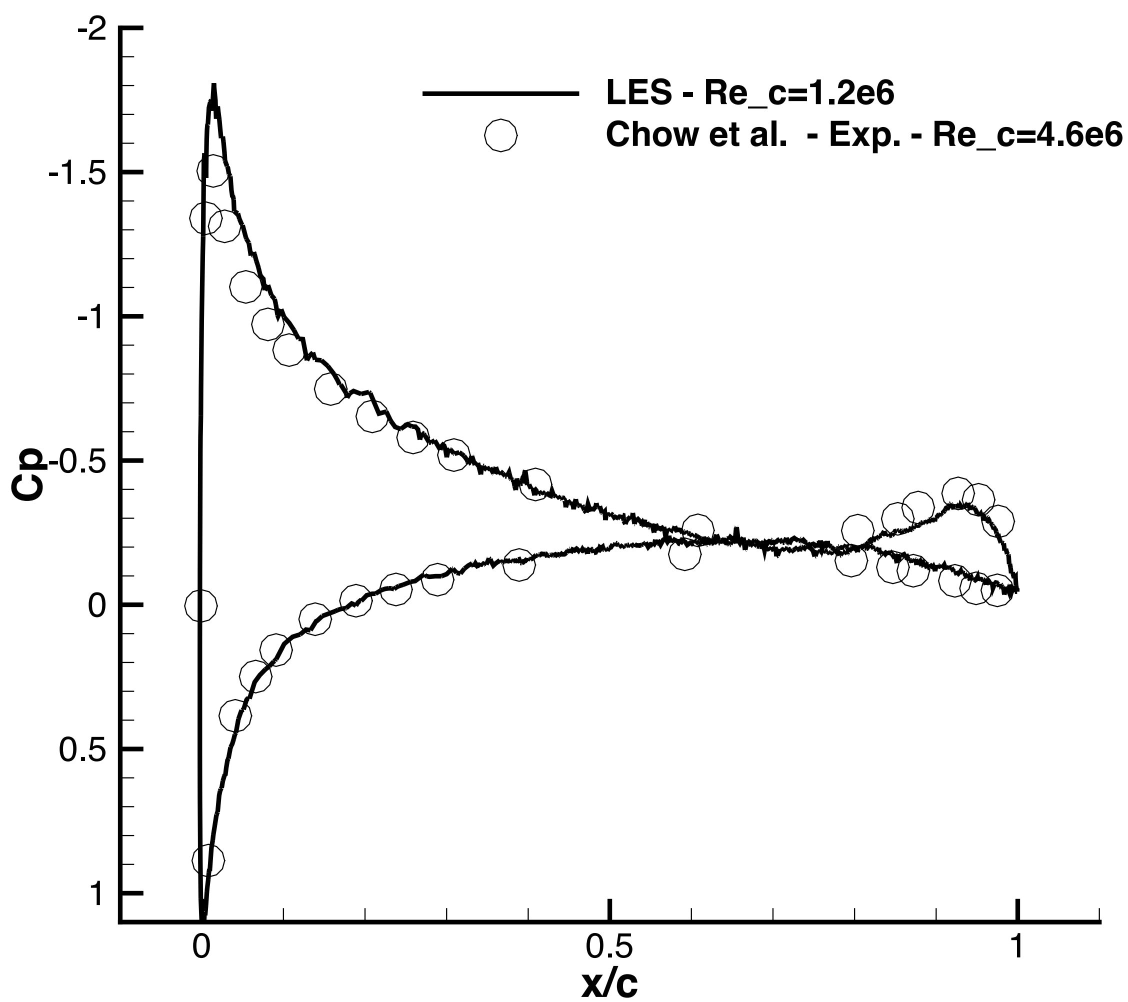
Existing simulations

Method	Global DOFs
RANS (modified Baldwin-Barth) Dacles-Mariani et al. (1997)	$2.5 \cdot 10^6$
RANS (Lag RST), Churchfield et al. (2013)	$13.8 \cdot 10^6$
LES, Uzun et al. (2006)	$26.2 \cdot 10^6$
ILES, Jiang et al. (2008)	$26 \cdot 10^6$
Present study	$\mathbf{15.3 \cdot 10^6}$

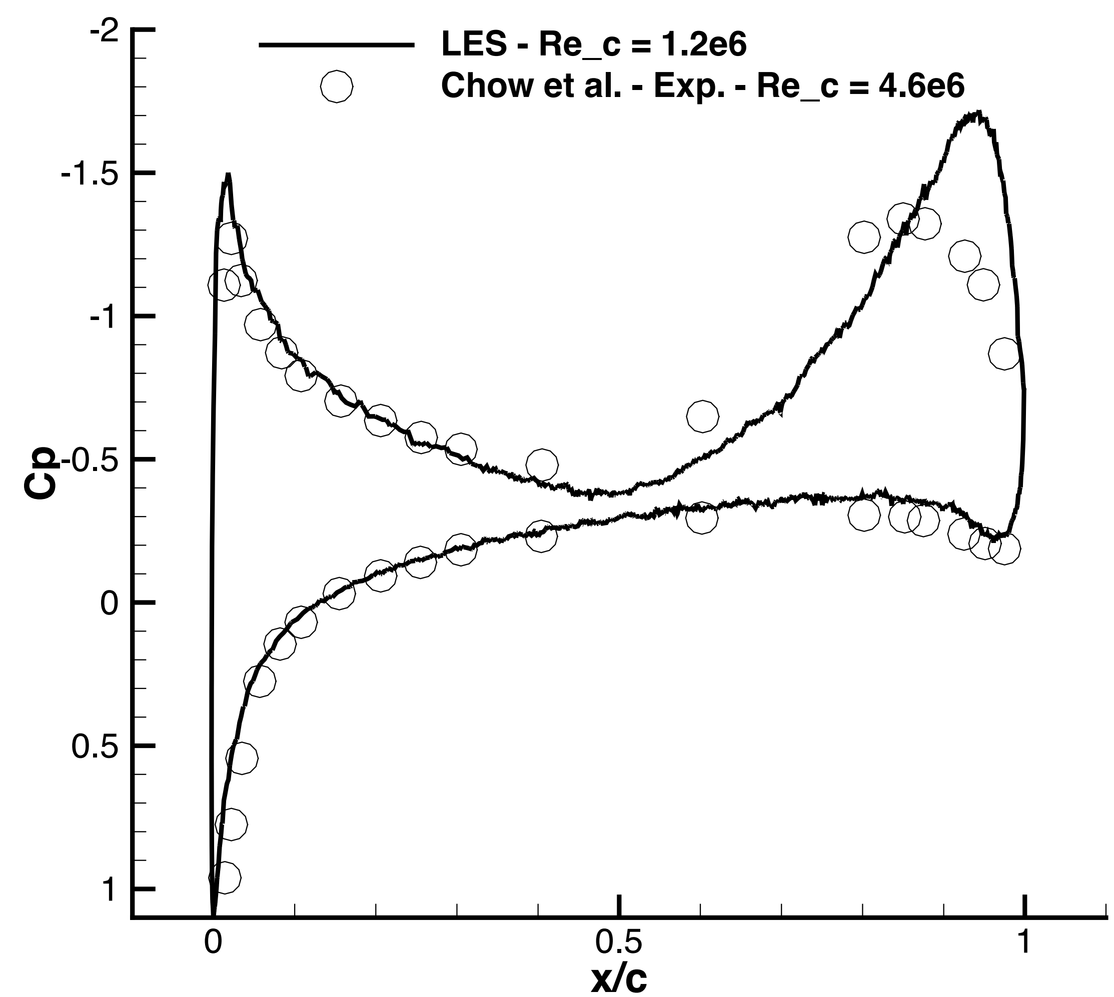
Pressure coefficient distribution



$z\,/\,b$ = 0.833



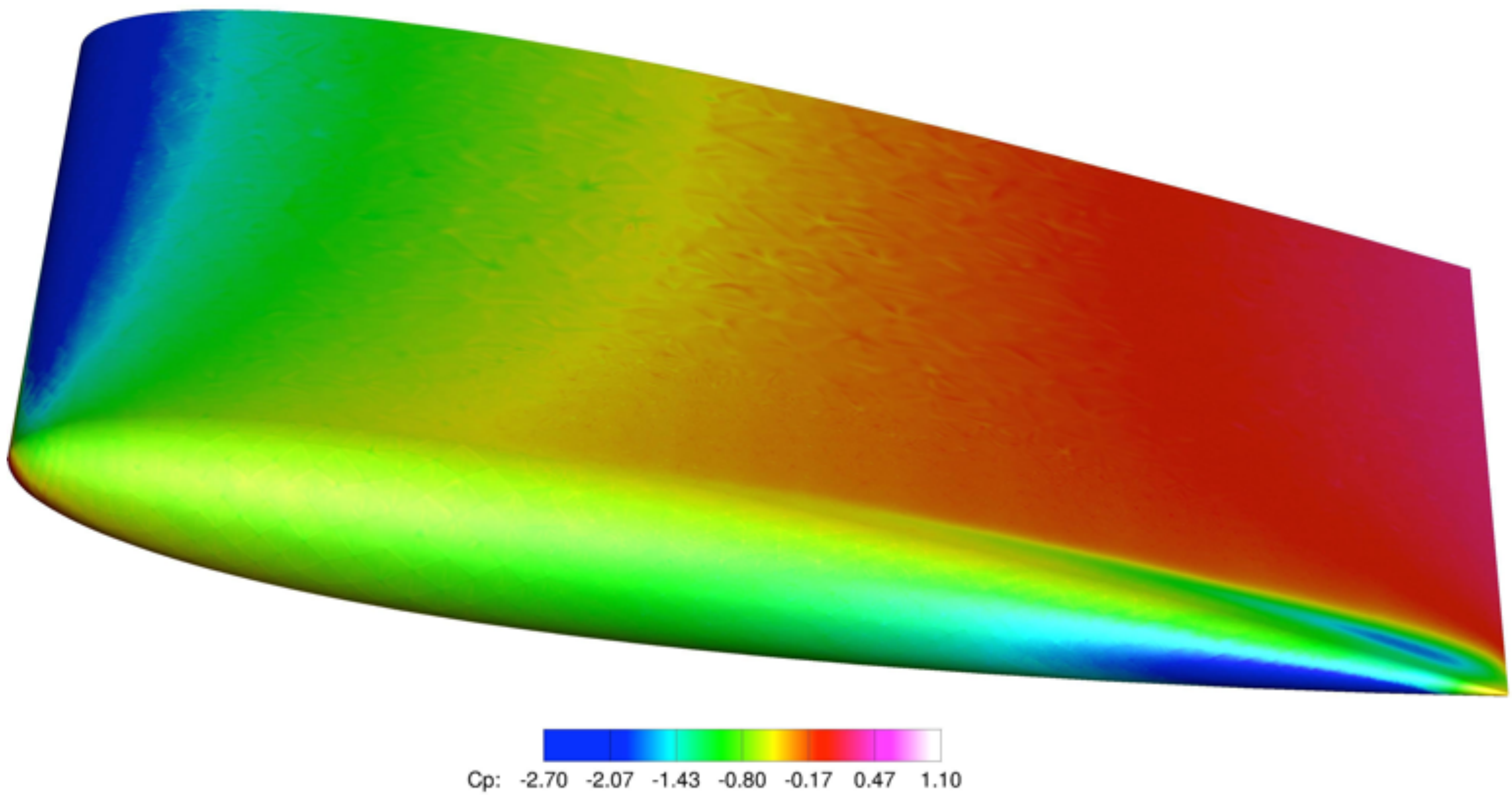
$z\,/\,b$ = 0.899



Transient simulation of a wingtip vortex at $Re_c = 1.2 \cdot 10^6$

Lombard, Moxey, Hoessler, Dhandapani, Taylor and Sherwin, under review in AIAA Journal

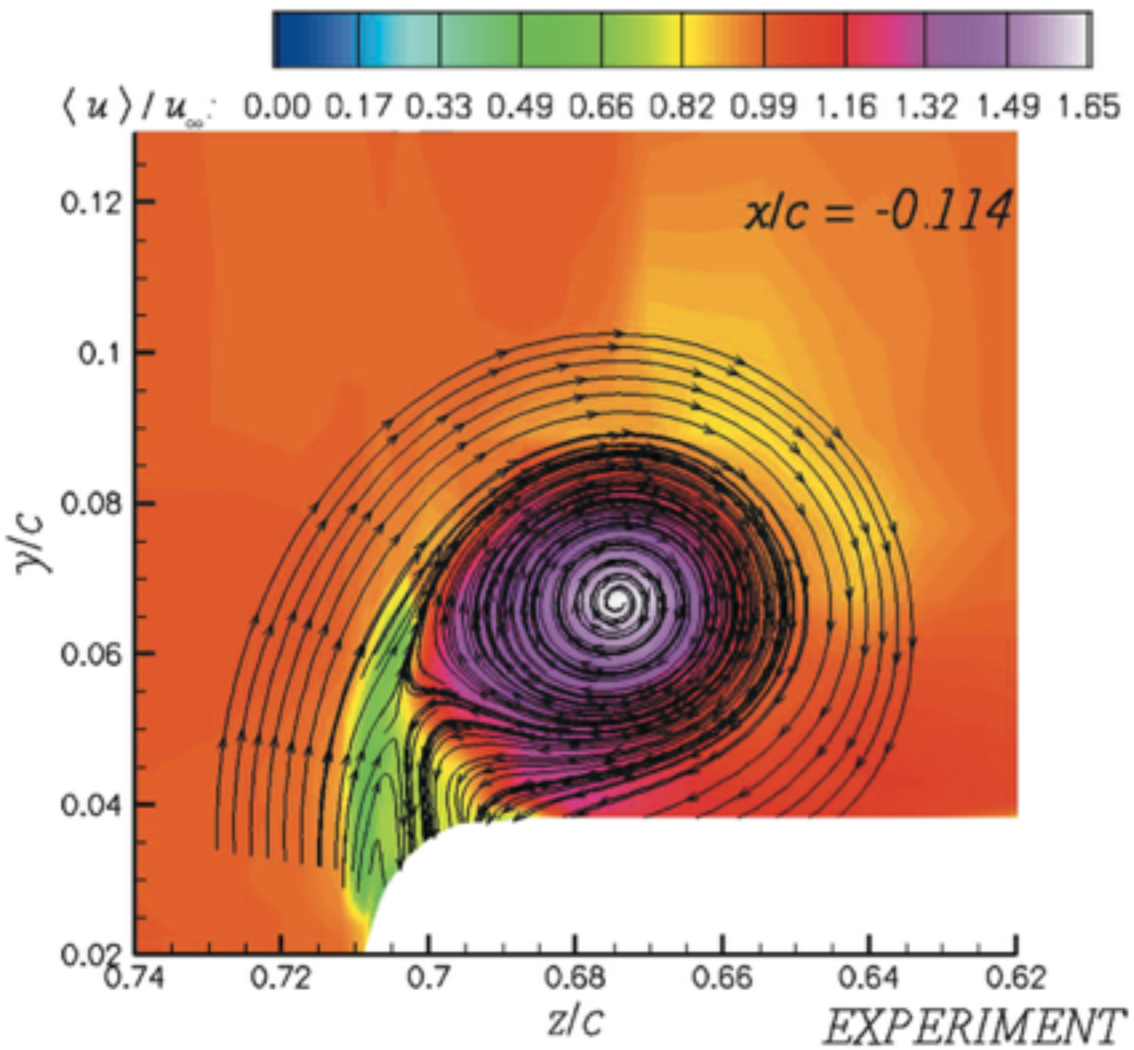
Pressure coefficient distribution



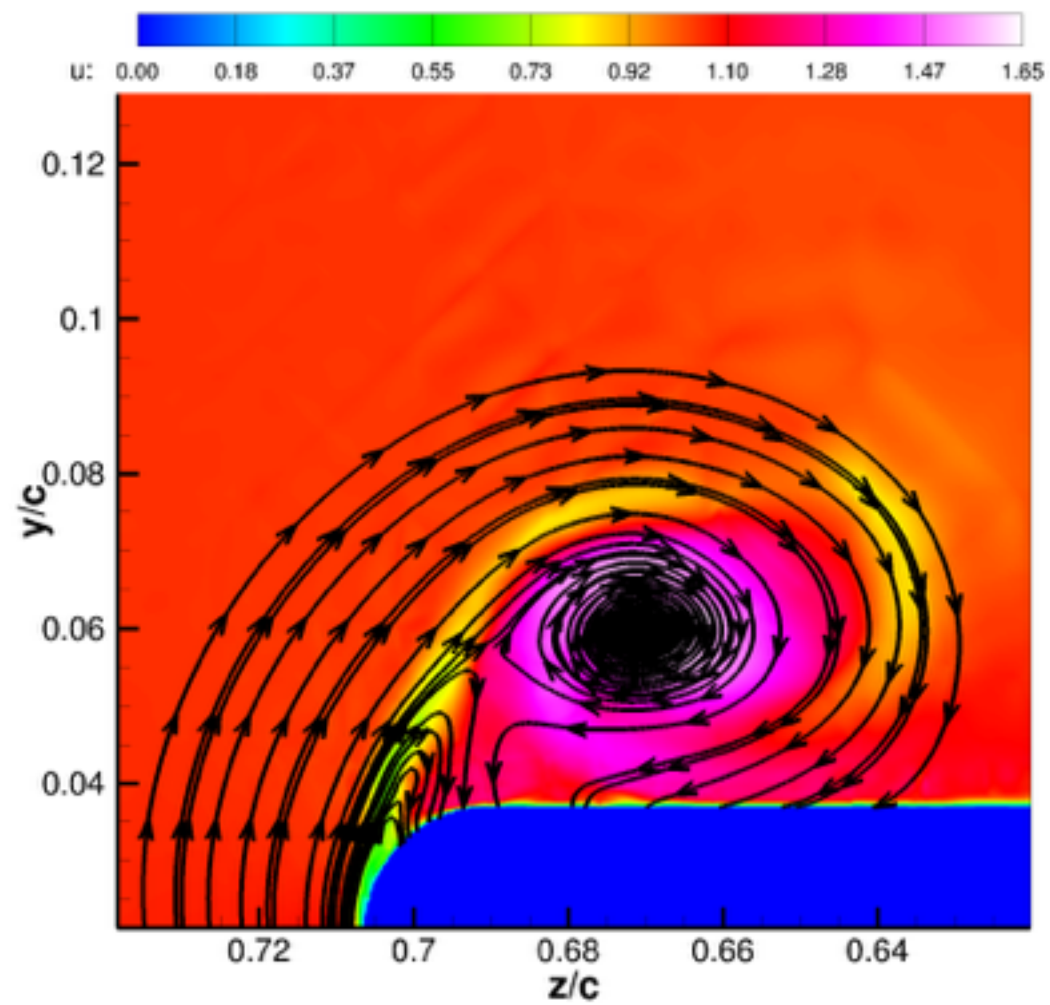
Time averaged distribution over 1 convective time unit

$t_c = tc/U_\infty$

Vortex core



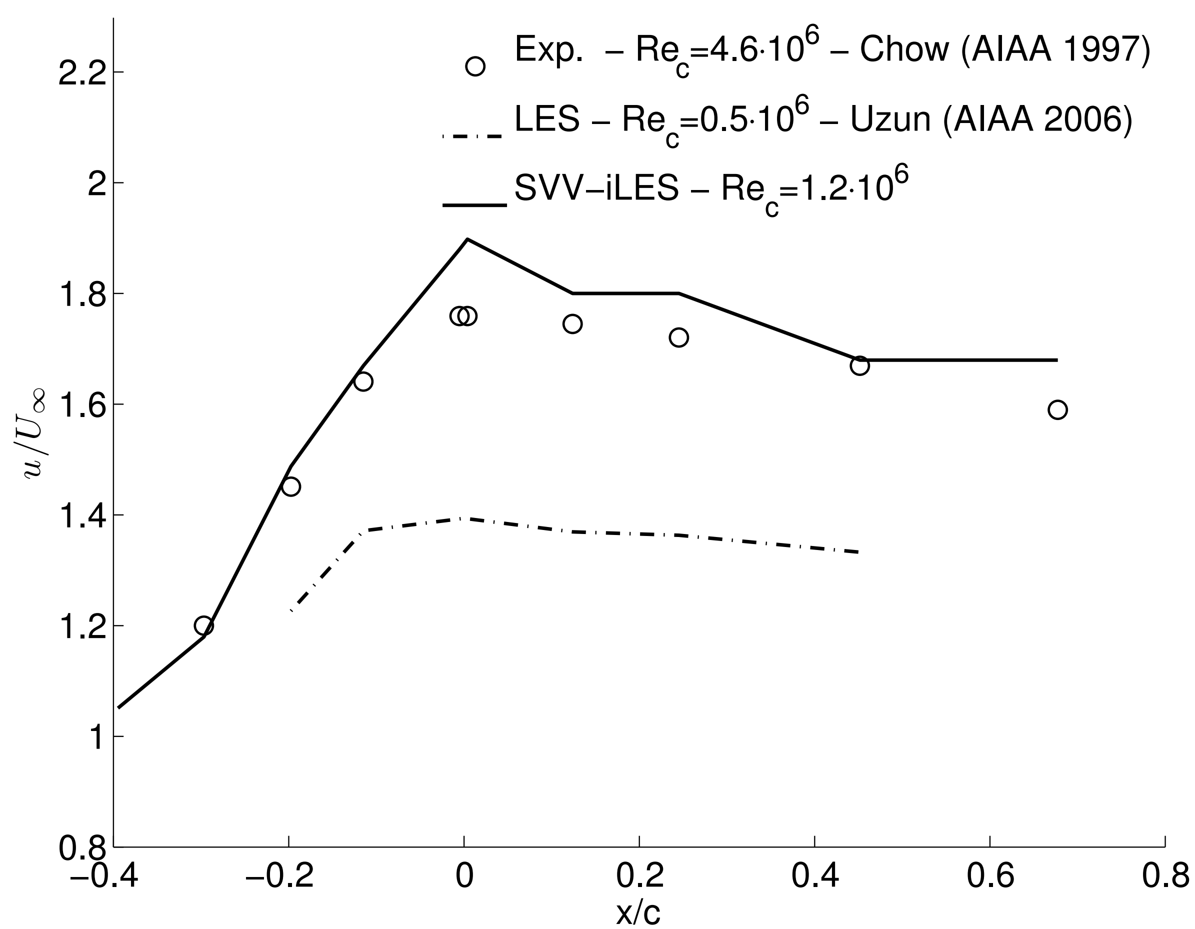
Experiment



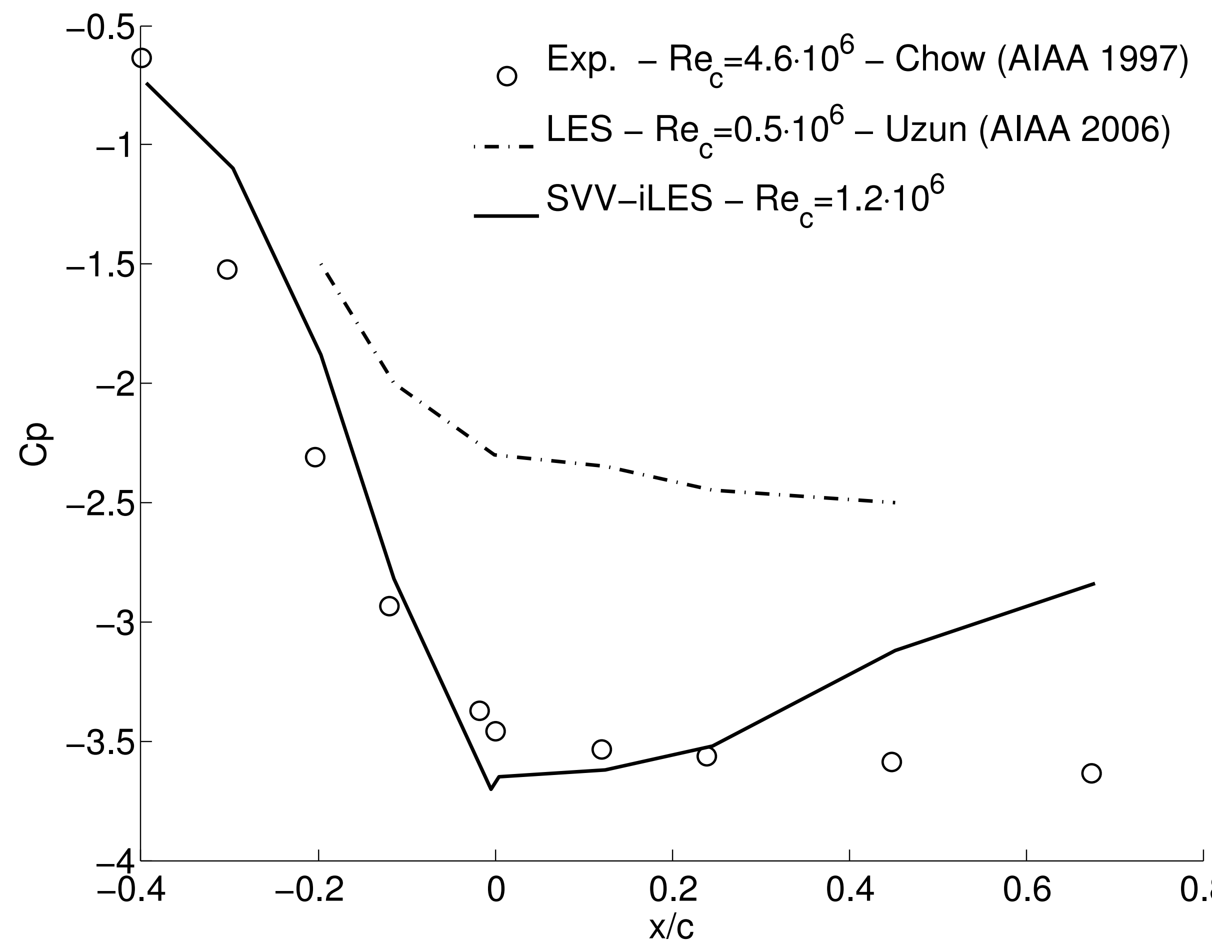
Simulation

$x / c = -0.114$

Vortex core



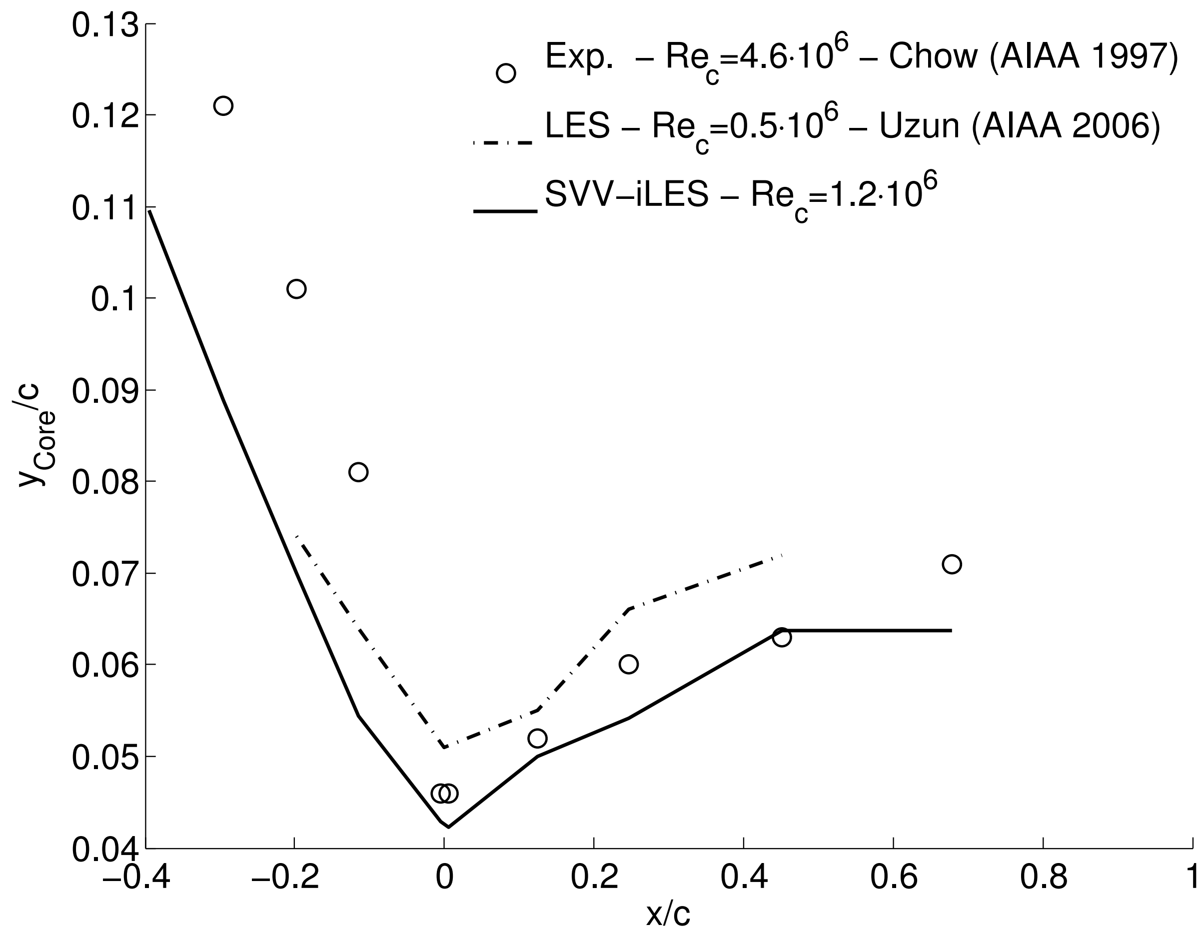
Axial velocity

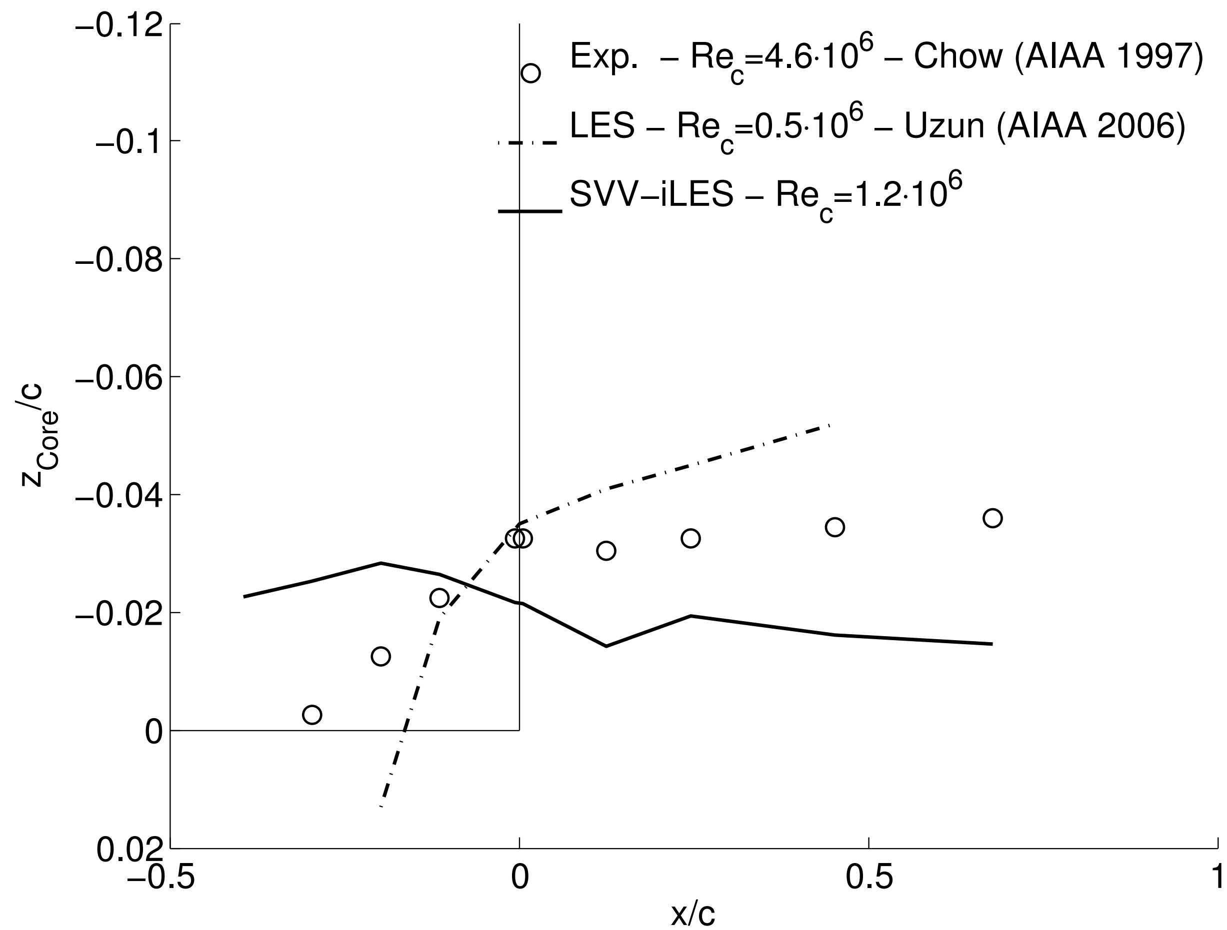


C_p distribution

Pretty good agreement

Vortex core location





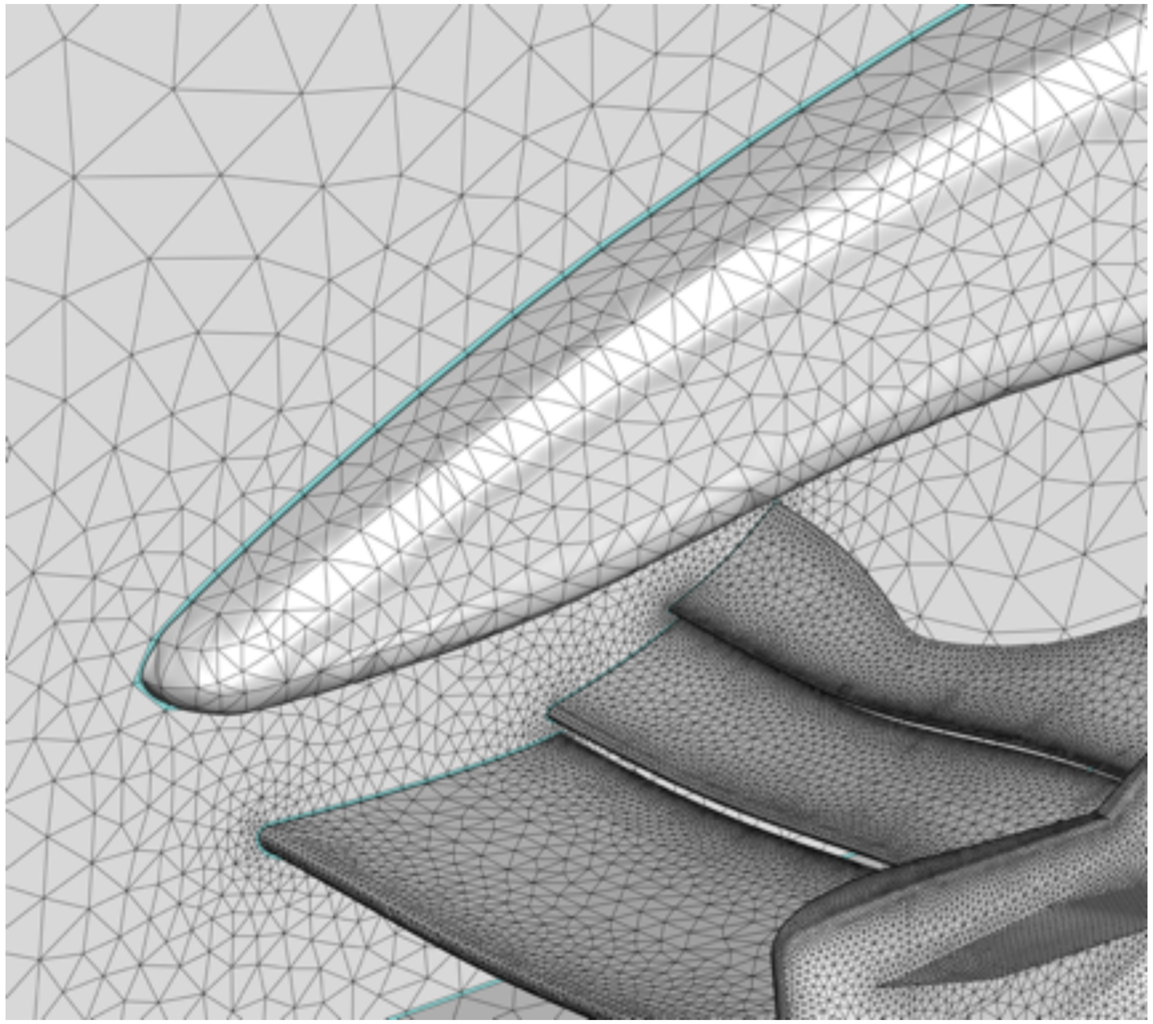
Above wing

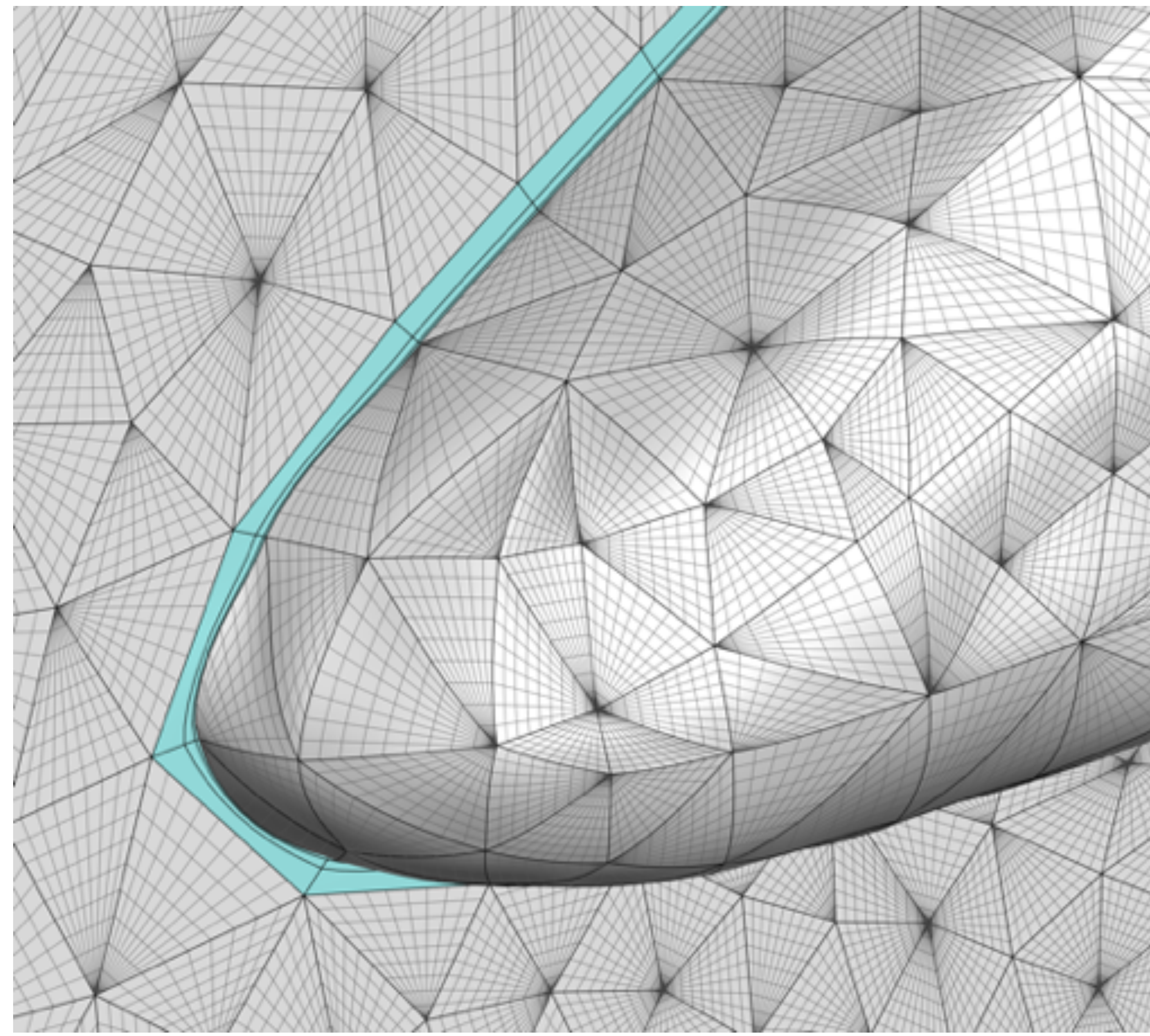
Spanwise location

Good agreement

Clearly not as good!

More complex geometry F1 front wing

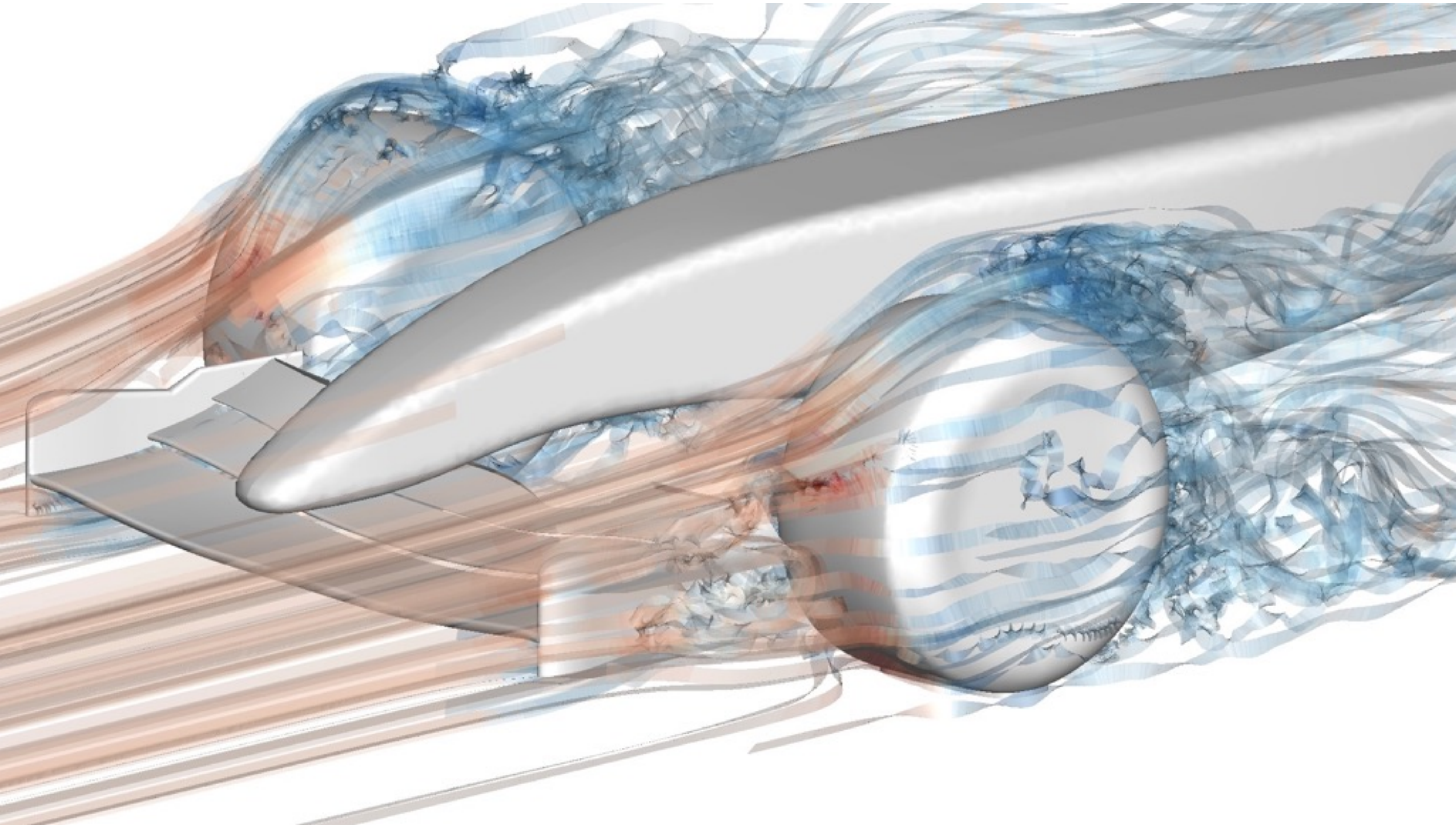




~1.75m elements
r = 3, 3-5 layers in BL
P = 3-5
~2-8k cores (ARCHER)
Re ~ 100k

An isoparametric approach to high-order curvilinear boundary-layer meshing
Moxey, Hazan, Sherwin, Peiro, under review in Comp. Meth. Appl. Mech. Eng.

Initial results



Conclusions

- High-order methods can be applied to these problems and successfully capture essential flow dynamics

- Still a need for high-order mesh generation strategies for coarse grid

- Promising results for larger and more complex geometries

- Next steps: improving contact patch boundary treatment, improve stabilisation techniques

- Many thanks to ARCHER for (lots of!) computing time under UKTC and resource allocation panels

Nektar++ high-order framework

Framework for spectral(/hp) element method:

- Dimension independent, supports CG/DG/HDG
- Mixed elements (quads/tris, hexes, prisms, tets, pyramids) using hierarchical modal and classical nodal formulations
- Solvers for (in)compressible Navier-Stokes, advection-diffusion-reaction, shallow water equations, ...
- Parallelised with MPI, tested scaling up to ~10k cores

http://www.nektar.info/

nektar-users@imperial.ac.uk

If you like high order methods...

- **MS103, MS128**: high- and low-order finite element software for the future
- Morning and afternoon sessions
- Room **151G**
- My talk: HDG vs. CG

Thanks for listening!

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