Applications of the spectral/hp element method to complex flow geometries

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Overview

• Motivation

• Challenges of transient flow simulations

• Results

• Future work
Motivation

Primary research goal is to investigate challenging external aerodynamics cases:

- High Reynolds numbers
- Complex three-dimensional geometries
- Large resolution requirements
- Transient dynamics
Motivation

• (Fully resolved) DNS gives extremely accurate results but is too expensive for high-Re simulations.

• How can we apply existing efficient academic DNS codes for industrial applications?

DNS of periodic hill
2D spectral element + 1D Fourier spectral
~25 million dof
NACA 0012 wing tip

Re = 4.5 \times 10^6

(Chow et al., AIAA Journal, 1997)

Difficult to capture transient effects with RANS
Flow characteristics

How low can we go in terms of resolution and still capture essential flow features?
High-order mesh generation

Boundary layer grids are hard to generate:

- High shear near walls
- First element needs to be of size roughly $O(Re^{-2})$
- Unfeasible to run with this number of elements in the entire domain and across surface of wall
- Therefore highly-stretched elements required
- Also has to be coarse for high-order to make sense
Isoparametric mapping

Shape function is a mapping from reference element (parametric coordinates) to mesh element (physical coordinates)

An isoparametric approach to high-order curvilinear boundary-layer meshing
Subdivide the reference element in order to obtain a boundary layer mesh.
NACA 0012 boundary layer grid

High order mesh
\[ P = 5 \]

After splitting
More complex transforms

Quads to triangles  Prisms to tetrahedra

On the generation of curvilinear meshes through subdivision of isoparametric elements
Stabilisation

Very high Reynolds numbers + under-resolution will inevitability cause instability. Common causes:

• Consistent integration of nonlinear terms

• Insufficient dissipation from the numerical method

Here we use

• Consistent integration of nonlinear terms

• Spectral vanishing viscosity
Navier-Stokes Solver

Navier–Stokes:
\[
\partial_t \mathbf{u} + \mathbf{N}(\mathbf{u}) = -\nabla p + \nu \nabla^2 \mathbf{u}
\]
\[
\nabla \cdot \mathbf{u} = 0
\]

Velocity correction scheme (aka stiffly stable):

Advection:
\[
\mathbf{u}^* = -\sum_{q=1}^{J} \alpha_q \mathbf{u}^{n-q} - \Delta t \sum_{q=0}^{J-1} \beta_q \mathbf{N}(\mathbf{u}^{n-q})
\]

Pressure Poisson:
\[
\nabla^2 p^{n+1} = \frac{1}{\Delta t} \nabla \cdot \mathbf{u}^*
\]

Helmholtz:
\[
\nabla^2 \mathbf{u}^{n+1} - \frac{\alpha_0}{\nu \Delta t} \mathbf{u}^{n+1} = -\frac{\mathbf{u}^*}{\nabla \Delta t} + \frac{1}{\nu} \nabla p^{n+1}
\]
Aliasing

Example: \( u(\xi) = \sum_{k=0}^{10} u_k \psi_i(\xi) \)

Galerkin projection of \( u^2 \) using:

- \( Q = 17 \) – exact Quadrature
- \( Q = 12 \) – sufficient for integrating 20th degree polynomials

Overview of nodal projection of $u^2$

$u(\xi) \in \mathcal{P}^P$

$I_{P \to Q}$

$f^P(\xi) = \tilde{u}^2(\xi) \in \mathcal{P}^P$

$GP_{Q \to P}$

$f^Q(\xi) = u^2(\xi) \in \mathcal{P}^{2P}$
Use tensor product structure

$P \times Q^2 + P^2 \times Q \Rightarrow O(P^3)$

Essentially performing sum factorisation

In 3D: $O(P^6)$ vs. $O(P^4)$

Dealiasing techniques for high-order spectral element methods on regular and irregular grids
Spectral Vanishing Viscosity

\[
\frac{\partial u}{\partial t} = \nu \nabla^2 u + S_{SVV}(u), \quad S_{SVV}(u) = \varepsilon \sum_{i=1}^{\text{dim}} \frac{\partial}{\partial x_i} \left[ Q_{\text{dim}} \star \frac{\partial u}{\partial x_i} \right]
\]

No SVV

\[
P_{\text{cut}} = 7, \quad \varepsilon_{SVV} = 0.1
\]

\[
P_{\text{cut}} = 3, \quad \varepsilon_{SVV} = 0.1
\]

NACA 0012 wing tip (Re = 1.2M)

Streamlines

Streamwise vorticity
## Existing simulations

<table>
<thead>
<tr>
<th>Method</th>
<th>Global DOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>RANS (modified Baldwin-Barth) Dacles-Mariani et al. (1997)</td>
<td>$2.5 \cdot 10^6$</td>
</tr>
<tr>
<td>RANS (Lag RST), Churchfield et al. (2013)</td>
<td>$13.8 \cdot 10^6$</td>
</tr>
<tr>
<td>LES, Uzun et al. (2006)</td>
<td>$26.2 \cdot 10^6$</td>
</tr>
<tr>
<td>ILES, Jiang et al. (2008)</td>
<td>$26 \cdot 10^6$</td>
</tr>
<tr>
<td>Present study</td>
<td>$15.3 \cdot 10^6$</td>
</tr>
</tbody>
</table>
Pressure coefficient distribution

$z / b = 0.833$

$z / b = 0.899$

Transient simulation of a wingtip vortex at $Re_c = 1.2 \cdot 10^6$
Lombard, Moxey, Hoessler, Dhandapani, Taylor and Sherwin, under review in AIAA Journal
Pressure coefficient distribution

Time averaged distribution over 1 convective time unit

\[ t_C = t_c/U_\infty \]
Vortex core

\[ \frac{x}{c} = -0.114 \]
Vortex core

Axial velocity

\[ \frac{u}{U_c} \]

\( x/c \)

\( -0.4 \) \( -0.2 \) \( 0 \) \( 0.2 \) \( 0.4 \) \( 0.6 \) \( 0.8 \)

\( 0.8 \) \( 1 \) \( 1.2 \) \( 1.4 \) \( 1.6 \) \( 1.8 \) \( 2 \) \( 2.2 \)

Exp. – \( Re_c = 4.6 \cdot 10^6 \) – Chow (AIAA 1997)
LES – \( Re_c = 0.5 \cdot 10^6 \) – Uzun (AIAA 2006)
___ SVV-iLES – \( Re_c = 1.2 \cdot 10^6 \)

\( C_p \) distribution

\( -0.4 \) \( -0.2 \) \( 0 \) \( 0.2 \) \( 0.4 \) \( 0.6 \) \( 0.8 \)

\( -4 \) \( -3.5 \) \( -3 \) \( -2.5 \) \( -2 \) \( -1.5 \) \( -1 \) \( -0.5 \)

Exp. – \( Re_c = 4.6 \cdot 10^6 \) – Chow (AIAA 1997)
LES – \( Re_c = 0.5 \cdot 10^6 \) – Uzun (AIAA 2006)
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Pretty good agreement
Vortex core location

Above wing
Good agreement

Spanwise location
Clearly not as good!
An isoparametric approach to high-order curvilinear boundary-layer meshing
Initial results
Conclusions

• High-order methods can be applied to these problems and successfully capture essential flow dynamics.

• Still a need for high-order mesh generation strategies for coarse grid.

• Promising results for larger and more complex geometries.

• Next steps: improving contact patch boundary treatment, improve stabilisation techniques.

• Many thanks to ARCHER for (lots of!) computing time under UKTC and resource allocation panels.
Nektar++ high-order framework

Framework for spectral(/hp) element method:

• Dimension independent, supports CG/DG/HDG

• Mixed elements (quads/tris, hexes, prisms, tets, pyramids) using hierarchical modal and classical nodal formulations

• Solvers for (in)compressible Navier-Stokes, advection-diffusion-reaction, shallow water equations, ...

• Parallelised with MPI, tested scaling up to ~10k cores

http://www.nektar.info/
nektar-users@imperial.ac.uk
If you like high order methods...

- **MS103, MS128**: high- and low-order finite element software for the future
- Morning and afternoon sessions
- Room **151G**
- My talk: HDG vs. CG
Thanks for listening!

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