A thermo-elastic analogy for high-order curvilinear meshing with control of mesh validity and quality

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Overview

- Motivation
- Linear elastic analogy
- Thermo-elastic formulation
- Results
- Conclusions

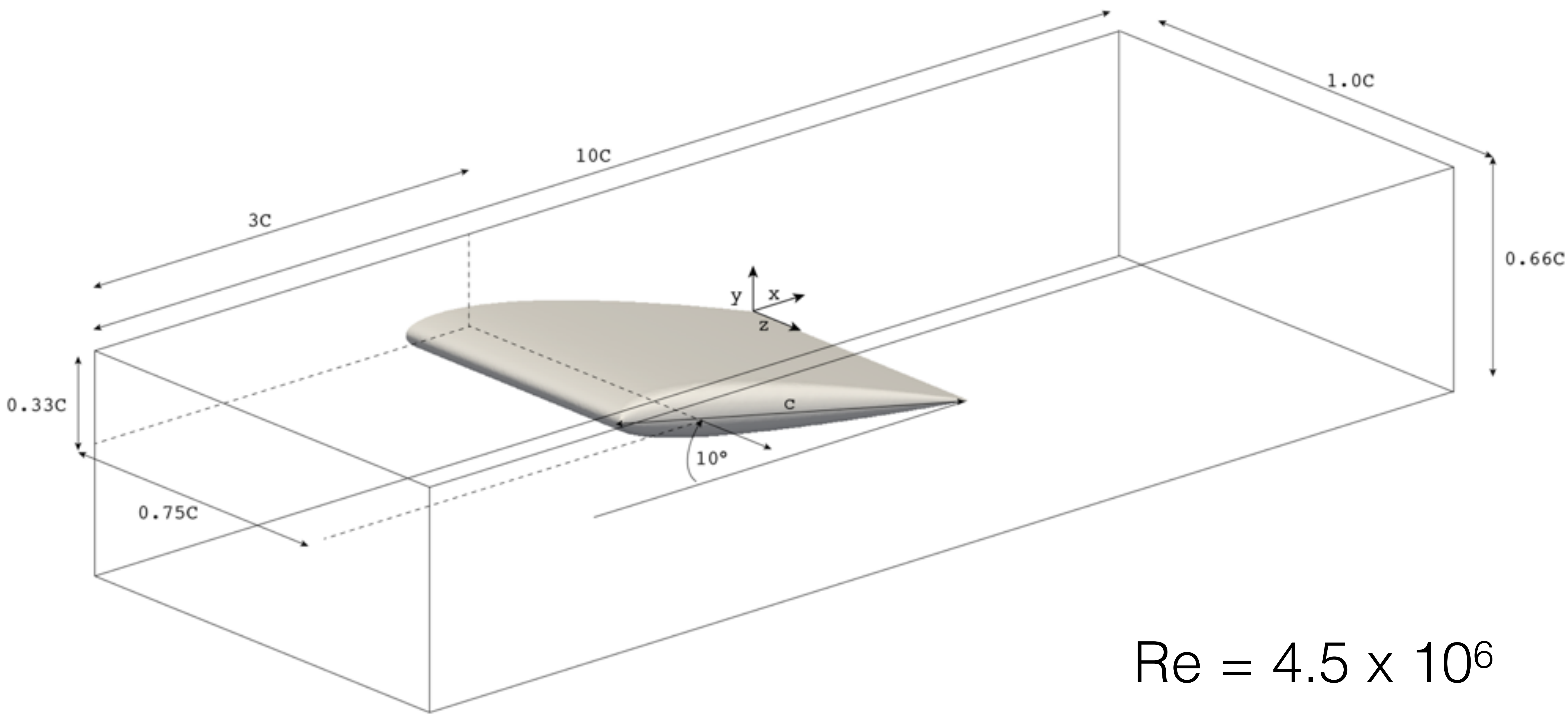
Motivation

Primary research goal is to investigate challenging external aerodynamics cases:

- High Reynolds numbers
- Complex three-dimensional geometries
- Large resolution requirements
- Transient dynamics

Using high-order spectral/*hp* element method.

NACA 0012 wing tip



Re = 4.5 x 10^6

Difficult to capture transient effects with RANS

Mesh generation strategy

1. Generate coarse linear grid (prismatic boundary, tetrahedral volume), typically O-type.

2. Apply surface curvature to prism faces

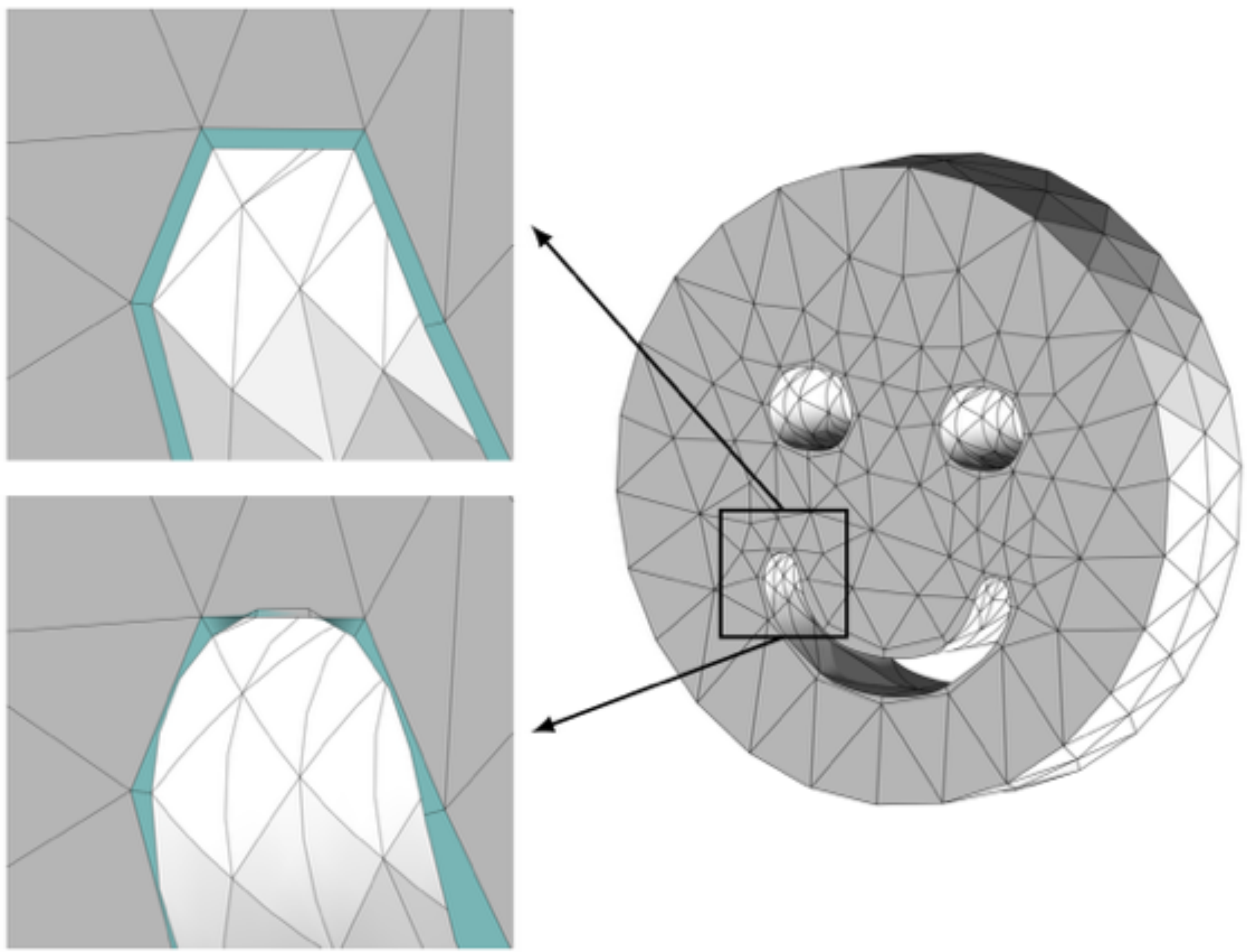
3. If it worked, split prisms to appropriate BL thickness using isoparametric mapping technique

4. Otherwise, repeat with thicker BL

Can suffer from robustness issues; curvature only on one face means element quality can be 'bad'.

High-order mesh generation

Curving mesh often leads to invalid elements



Grid deformation approach

- Start with a linear grid of a domain, which we consider to be a solid body
- Apply a deformation to the boundary which deforms edges and faces so that they align with the geometry
- Solve some equations to "push" curvature into the interior elements and (hopefully) prevent self-intersection

Linear elastic analogy

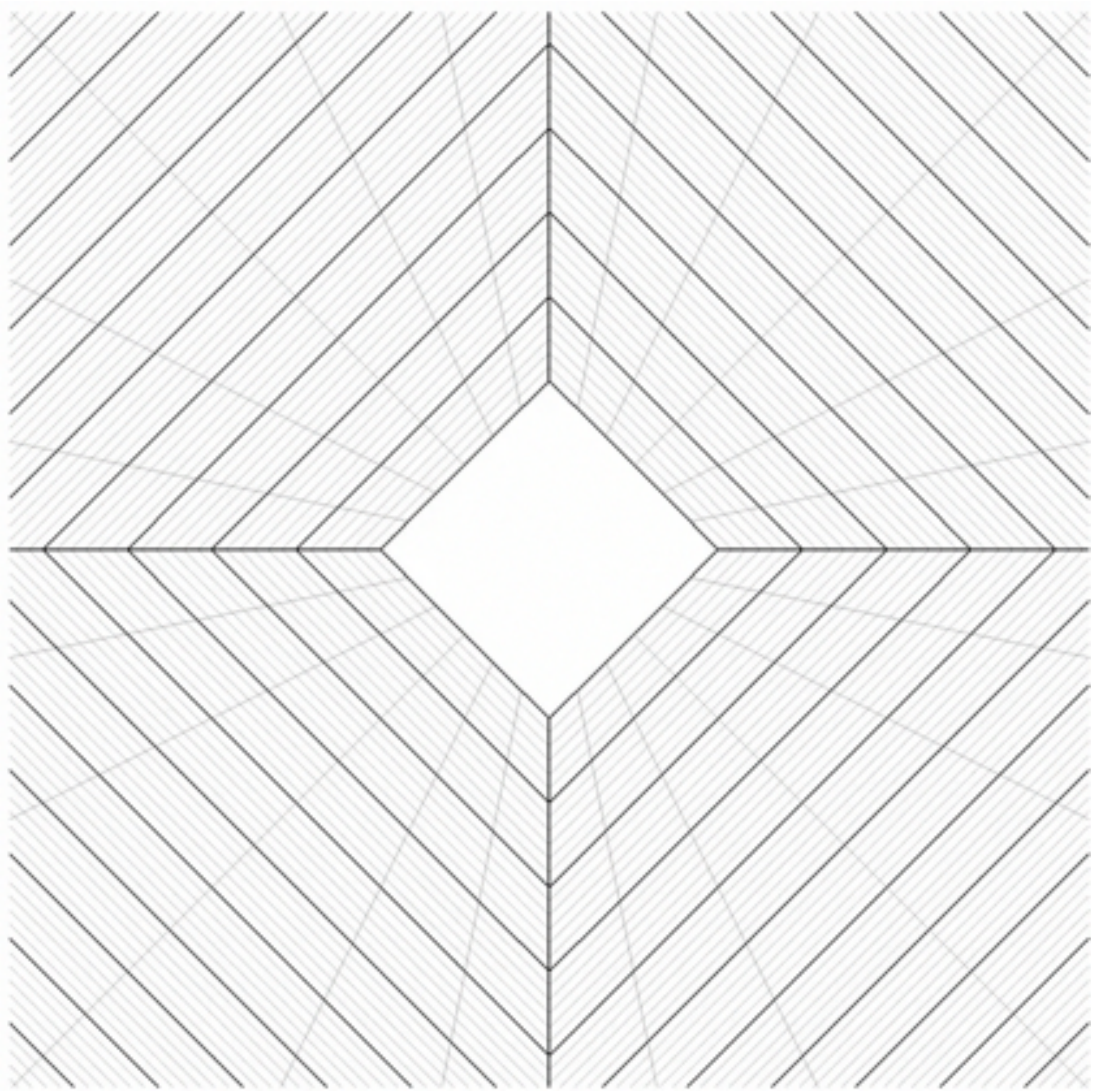
We use linear elasticity equations for displacement **u**

$$\nabla \cdot \mathbf{S} + \mathbf{f} = \mathbf{0} \quad \text{in } \Omega \qquad \mathbf{S} = \lambda \mathrm{Tr}(\mathbf{E})\,\mathbf{I} + \mu \mathbf{E}$$

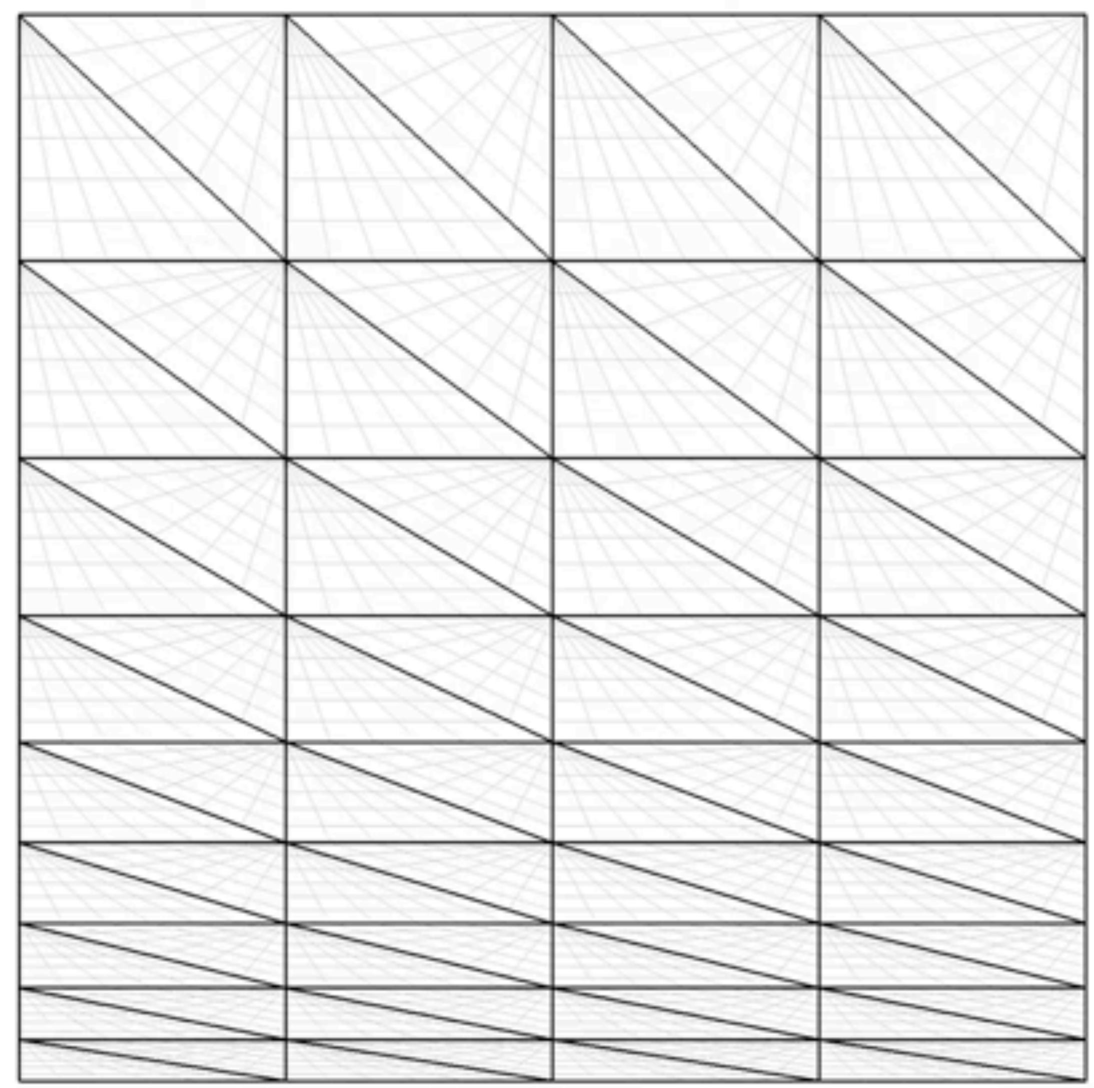
$$\mathbf{u} = \mathbf{g} \quad \text{in } \partial\Omega \qquad \mathbf{E} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)$$

- Assumes small deformations
- We can split up large deformations into a sequence of smaller steps to increase robustness
- Non-linear deformation is better but more expensive
- Quality of elements not guaranteed to be 'good'

Elastic analogy in action



Coarse quad grid to circle



Boundary layer grid

Thermal stress

- **Cheap modification:** add another analogy.
- Suppose that there are not just elastic stresses but thermal stresses.
- **Idea:** as the elements become more deformed, they 'heat up' and the effect of elasticity becomes less pronounced.
- Larger deformations are therefore permitted since elements shrink to fit surrounding deformation.

Formulation

- Assume stress tensor **S** can now be written as

$$\mathbf{S} = \mathbf{S}_e + \mathbf{S}_t$$

where $\mathbf{S}_t$ represents thermal stress term.

- The simplest model is that of a linear isotropic material so that

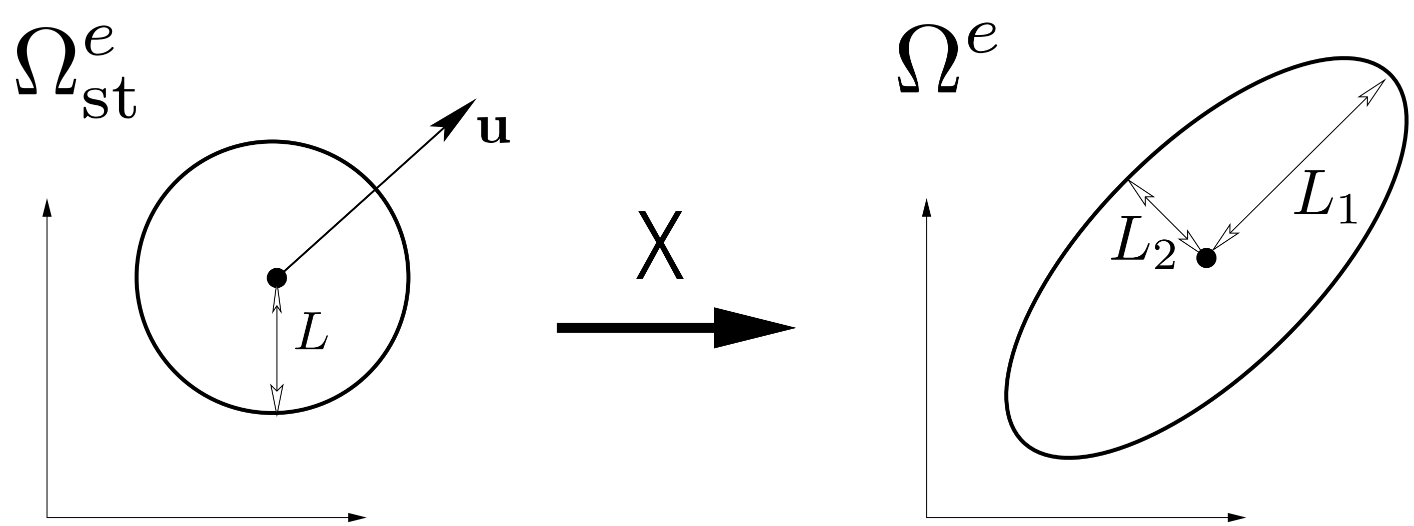
$$\mathbf{S}_\mathrm{t} = \beta(T - T_0)\mathbf{I}$$

where T is the temperature, T_0 is the temperature of the stress-free state, and β controls the amount of thermal stress.

- Only assumption is that $\mathbf{S}_t$ does not depend on displacement **u**.

Alternative thermal stresses

- Another stress form comes from high order mapping.
- Given a high-order mapping χ from standard region to element, we have a Jacobian matrix **J** and associated metric tensor $\mathbf{G} = \mathbf{J}^T\mathbf{J}$.
- Notionally **G** gives a description of the principal directions of deformation.



Alternative thermal stress

- Given an eigenvalue decomposition **G** = **PDP**$^{-1}$ we scale **D** according to stresses

$$e_i = \frac{L_i - L}{L} = \sqrt{\lambda_i} - 1$$

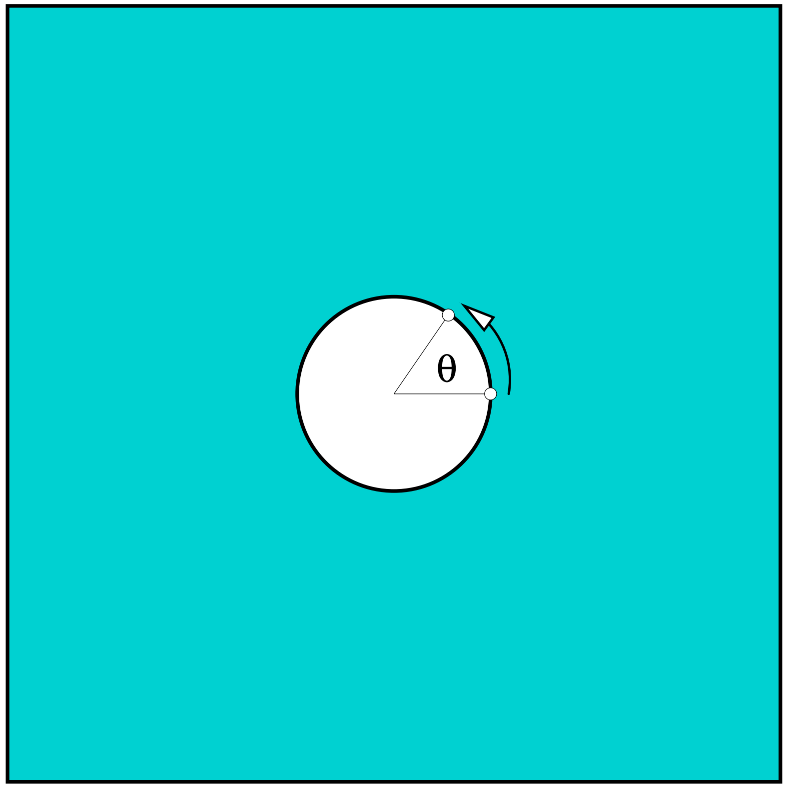
- Then to counteract the elasticity forces we take

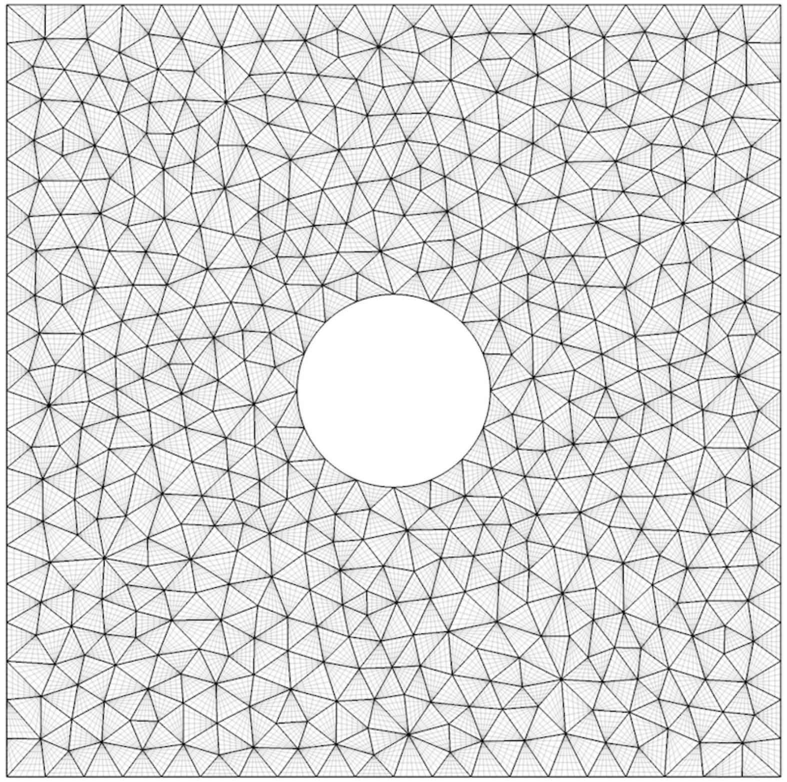
$$\mathbf{S}_t = -\beta\,\mathbf{P}^{-1}\,\mathrm{diag}\{e_1, e_2\}\,\mathbf{P}^{-1}$$

- where β again controls the amount of stress to add.

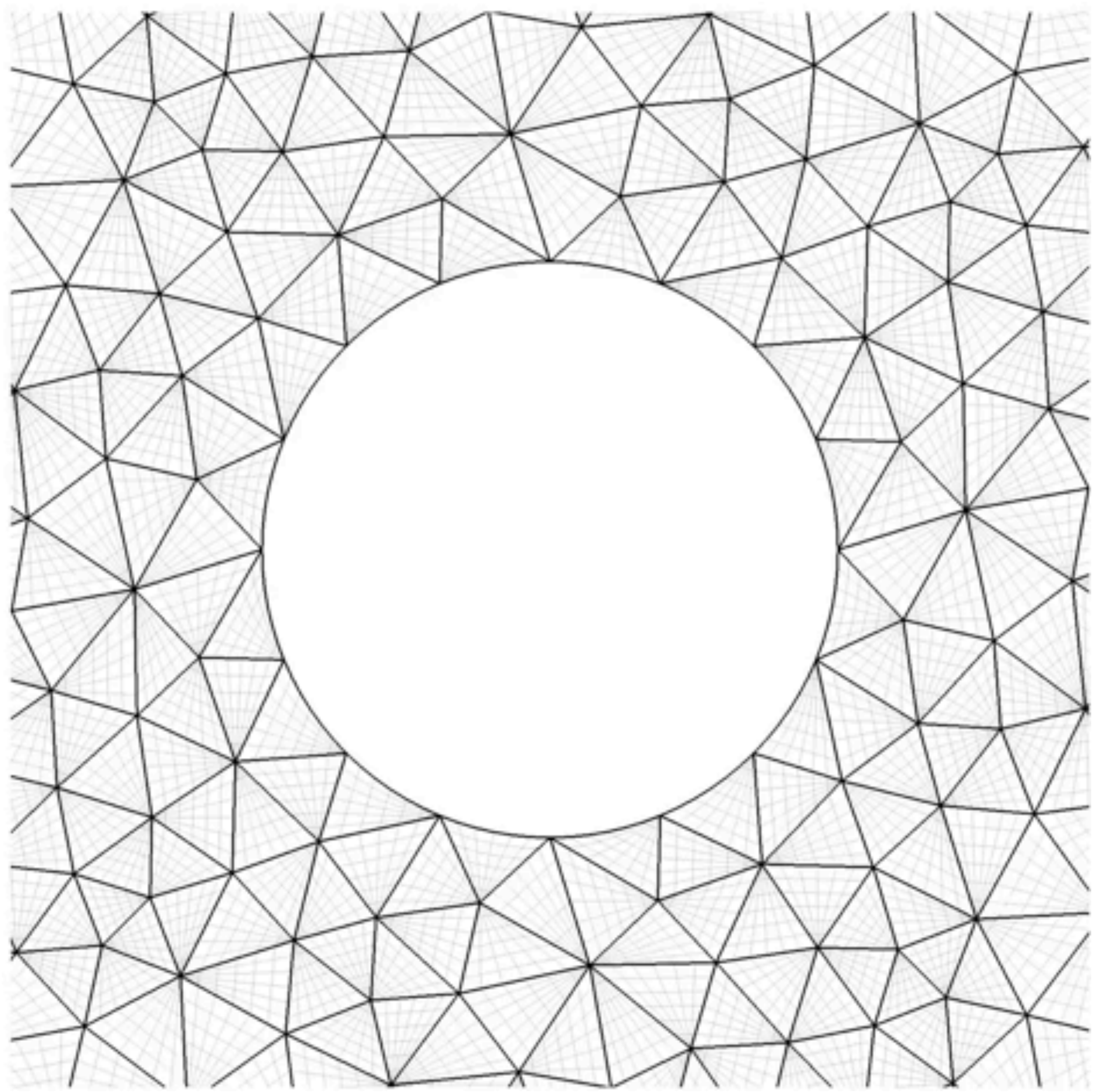
Test case

- Unstructured triangular mesh of circle inside square.
- Rotate circle until occurrence of negative Jacobian determinant at oversampled points
- Repeatedly apply equations in 1° increments, observe maximum rotation angle θ_{max} as a function of β.





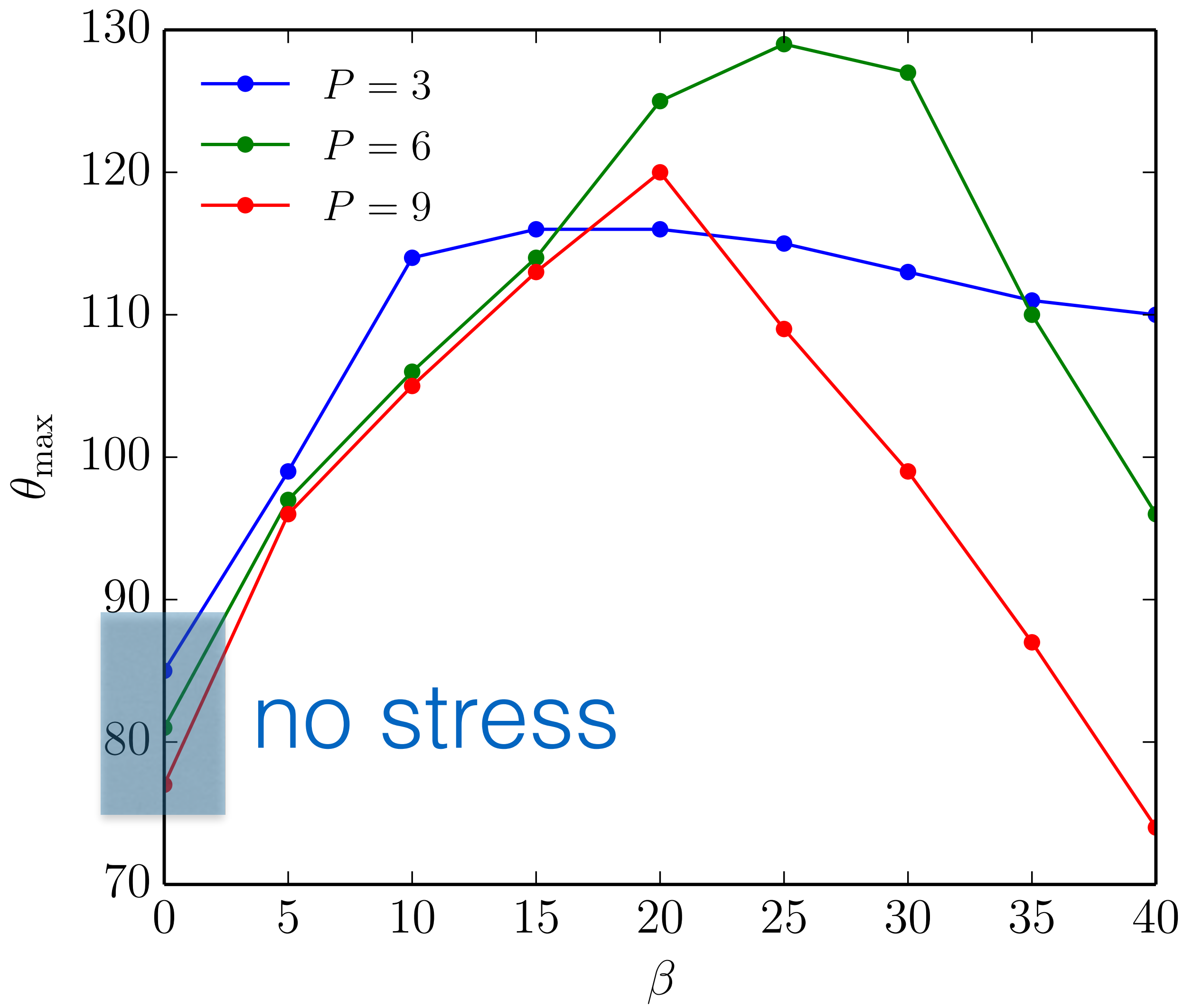
Test case results

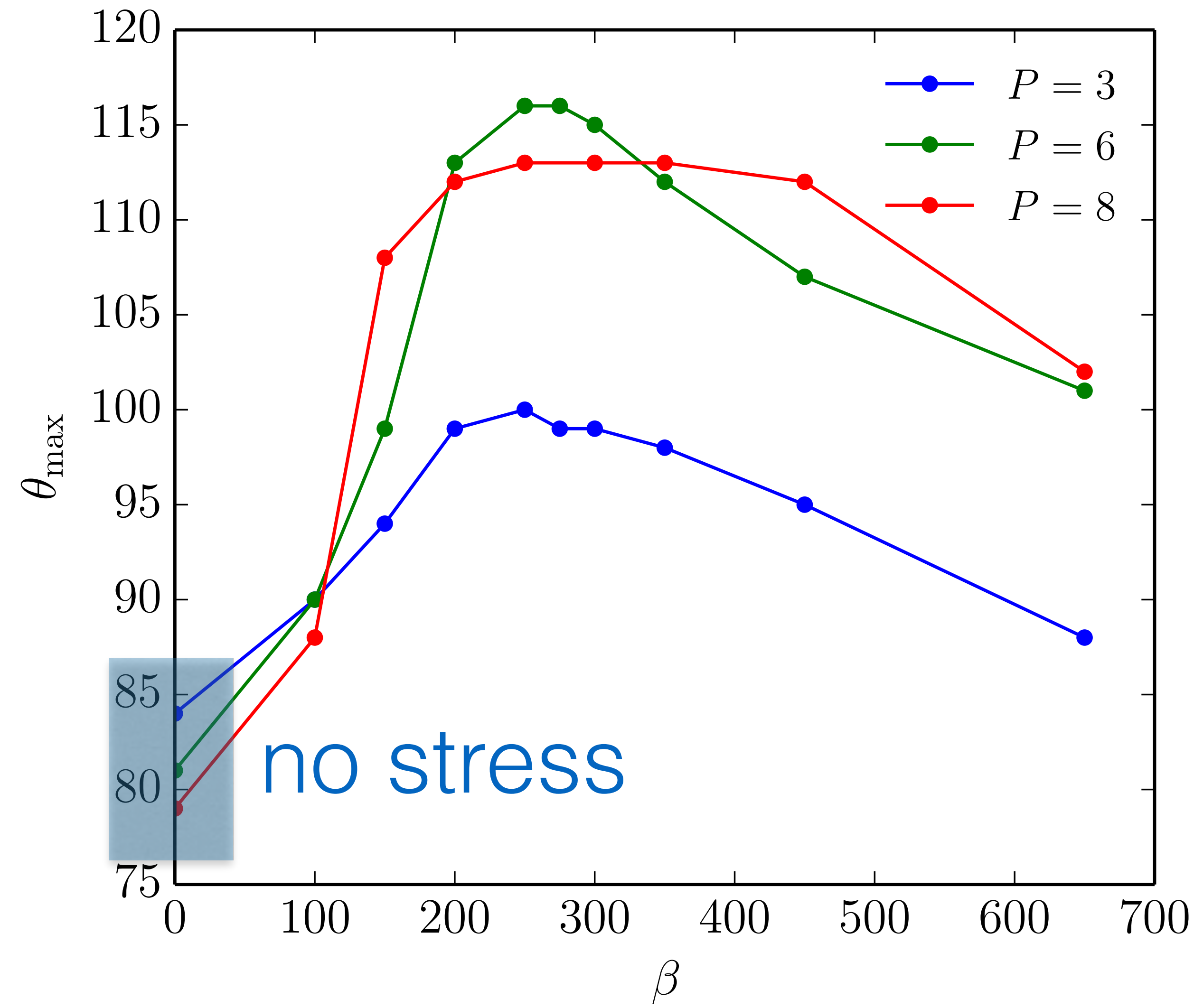


$P = 6$

Test case results

E = 1, v = 0.3, **f** = **0**





Isotropic stress

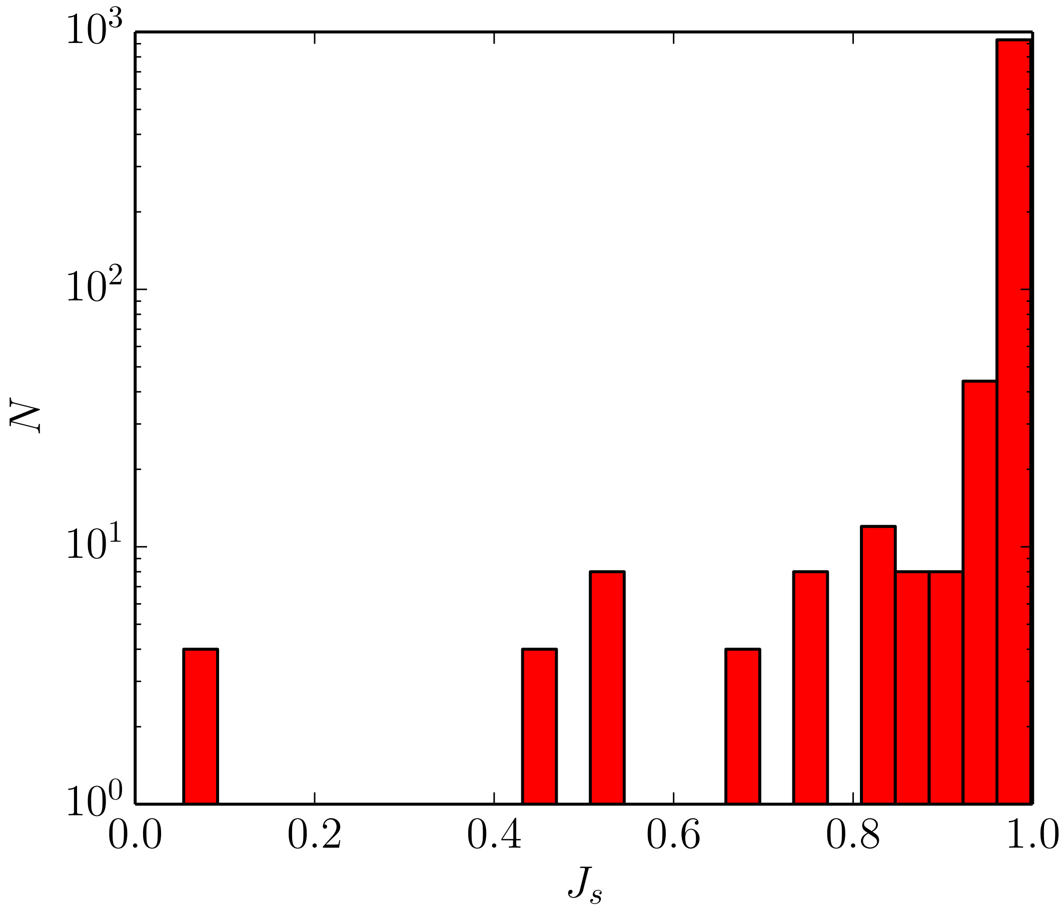
$\mathbf{S}_\mathrm{t} = \beta J\,\mathbf{I}$

Anisotropic stress

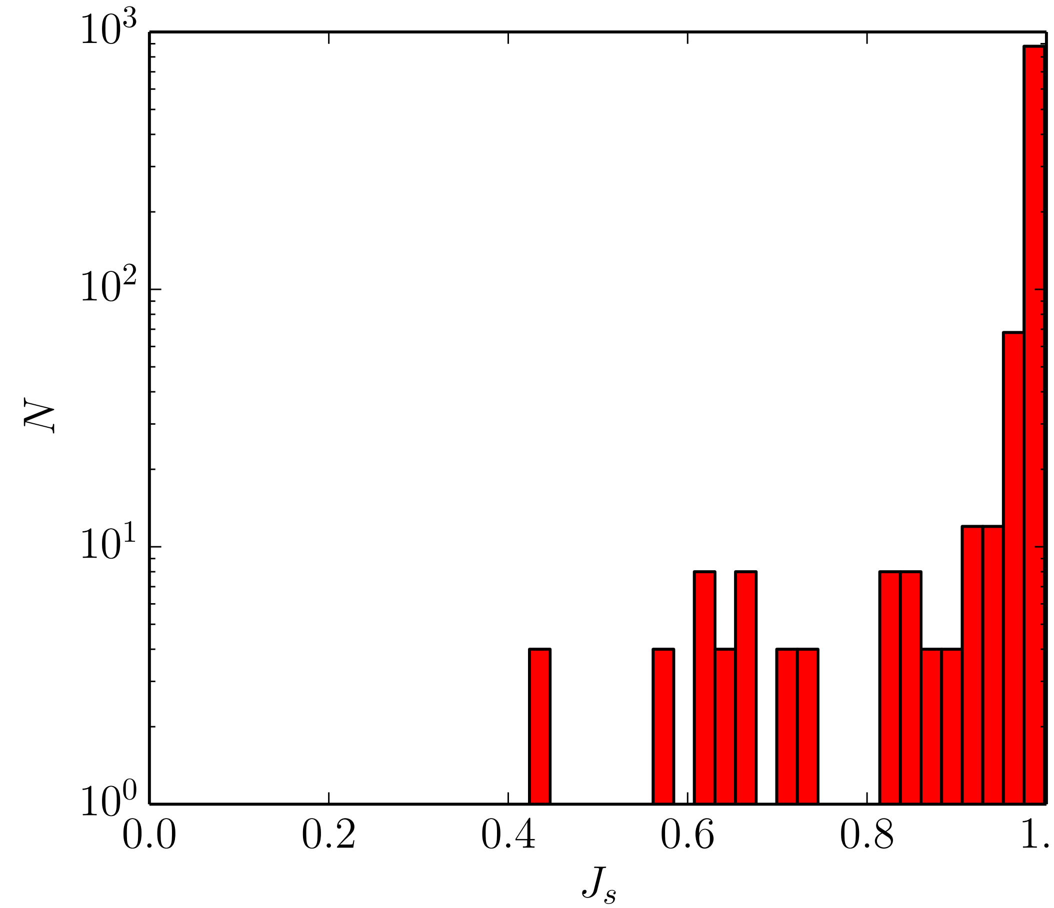
$\mathbf{S}_t = -\beta\,\mathbf{P}^{-1}\,\mathrm{diag}\{e_1, e_2\}\,\mathbf{P}^{-1}$

Effect on quality

measure element distortion $J_s = \dfrac{\min J(\xi)}{\max J(\xi)}$



without thermal stress



with thermal stress

Conclusions

- The incorporation of thermal stresses offers control on the validity of the mesh and improves the quality under large displacements of the boundary.

- Promising results for larger and more complex geometries.

- Work in three dimensions ongoing (solver implementation completed, temperature terms underway).

Nektar++ high-order framework

Linear elastic solver implemented using **Nektar++**:

- High-order spectral/hp element framework
- Dimension independent, supports CG/DG/HDG
- Mixed elements (quads/tris, hexes, prisms, tets, pyramids) using hierarchical modal and classical nodal formulations
- Solvers for (in)compressible Navier-Stokes, advection-diffusion-reaction, shallow water equations, **linear elasticity**, ...
- Parallel, scales up to ~10k cores

http://www.nektar.info/ nektar-users@imperial.ac.uk

Thanks for listening!