Utilising high-order direct numerical simulation for transient aeronautics problems

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Overview

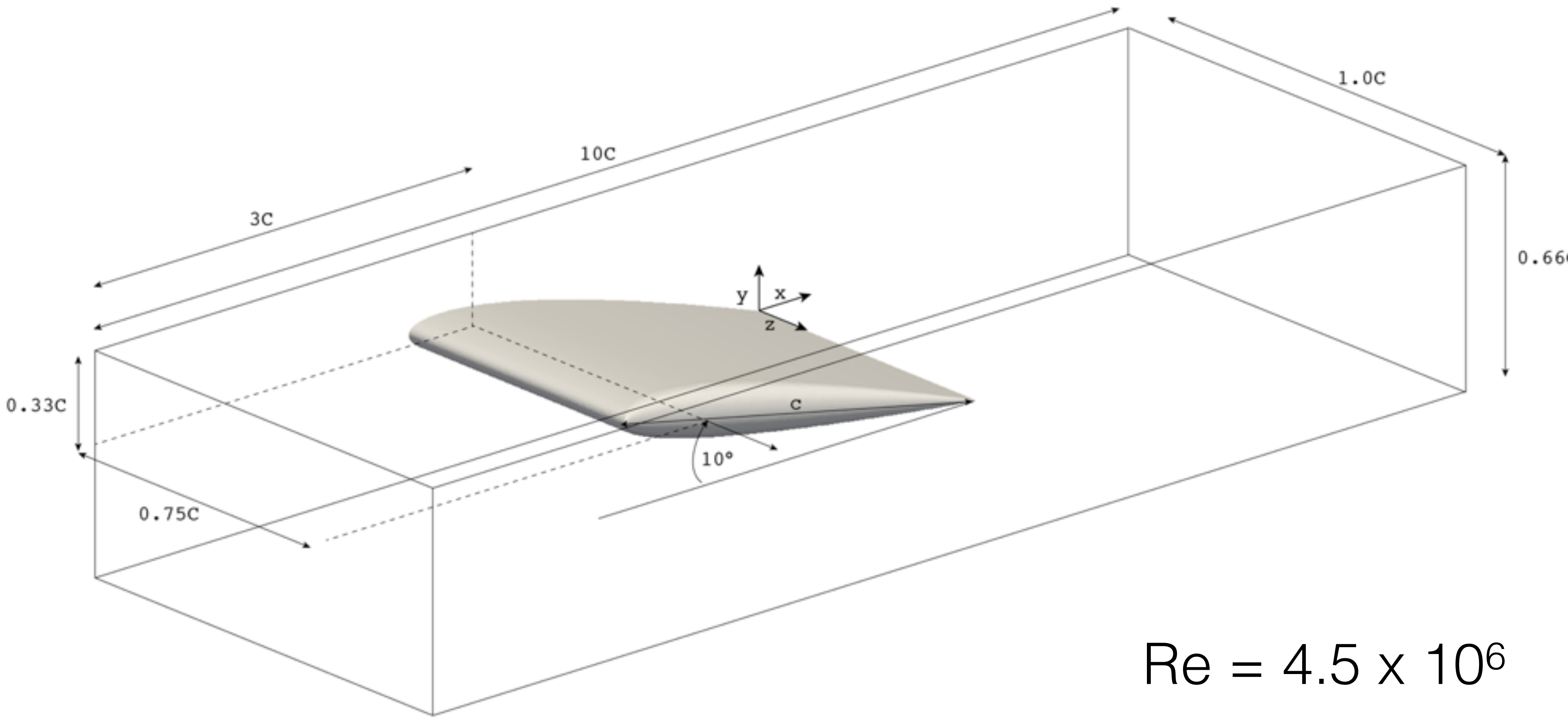
- Motivation
- Challenge: mesh generation
- Challenge: stabilisation
- Some results

Motivation

Primary research goal is to investigate challenging external aerodynamics cases:

- High Reynolds numbers
- Complex three-dimensional geometries
- Large resolution requirements
- Transient dynamics

NACA 0012 wing tip

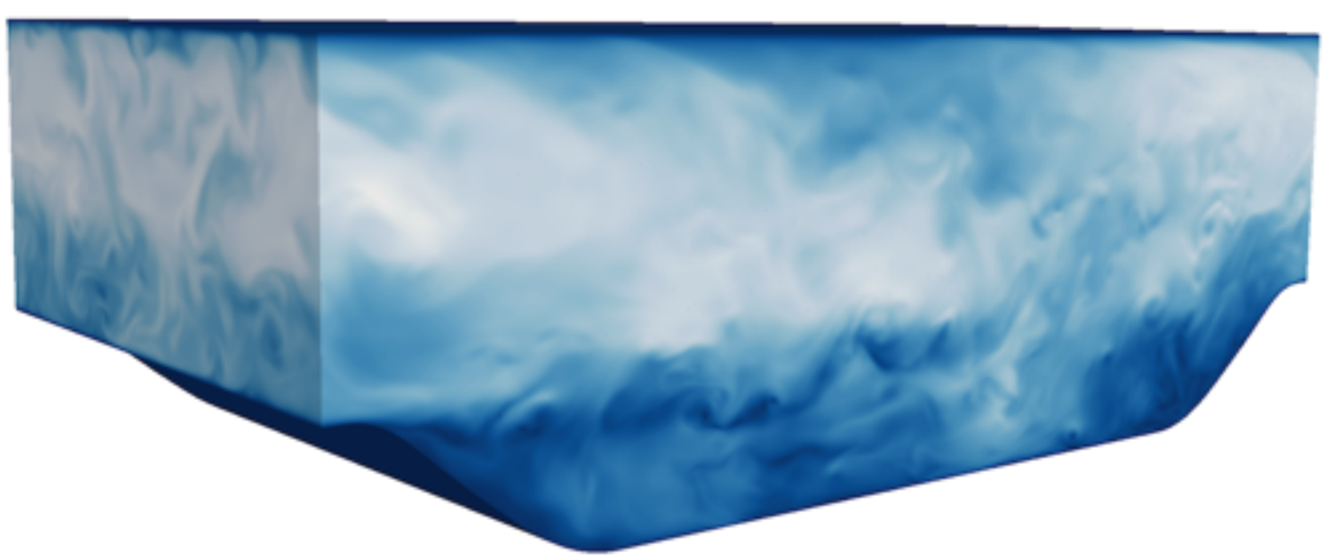


Re = 4.5 x 10^6

Difficult to capture transient effects with RANS

Motivation

- (Fully resolved) DNS gives extremely accurate results but is too expensive for these applications.
- How can we apply existing efficient academic DNS codes for industrial applications?



DNS of periodic hill
2D spectral element +
1D Fourier spectral
~25 million dof
quite expensive!

Nektar++ high-order framework

Framework for spectral(/hp) element method:

- Dimension independent, supports CG/DG/HDG
- Mixed elements (quads/tris, hexes, prisms, tets, pyramids) using hierarchical modal and classical nodal formulations
- Solvers for (in)compressible Navier-Stokes, advection-diffusion-reaction, shallow water equations, ...
- Parallelised with MPI, scales up to ~10k cores

http://www.nektar.info/
nektar-users@imperial.ac.uk

Challenge: mesh generation

Three stage process

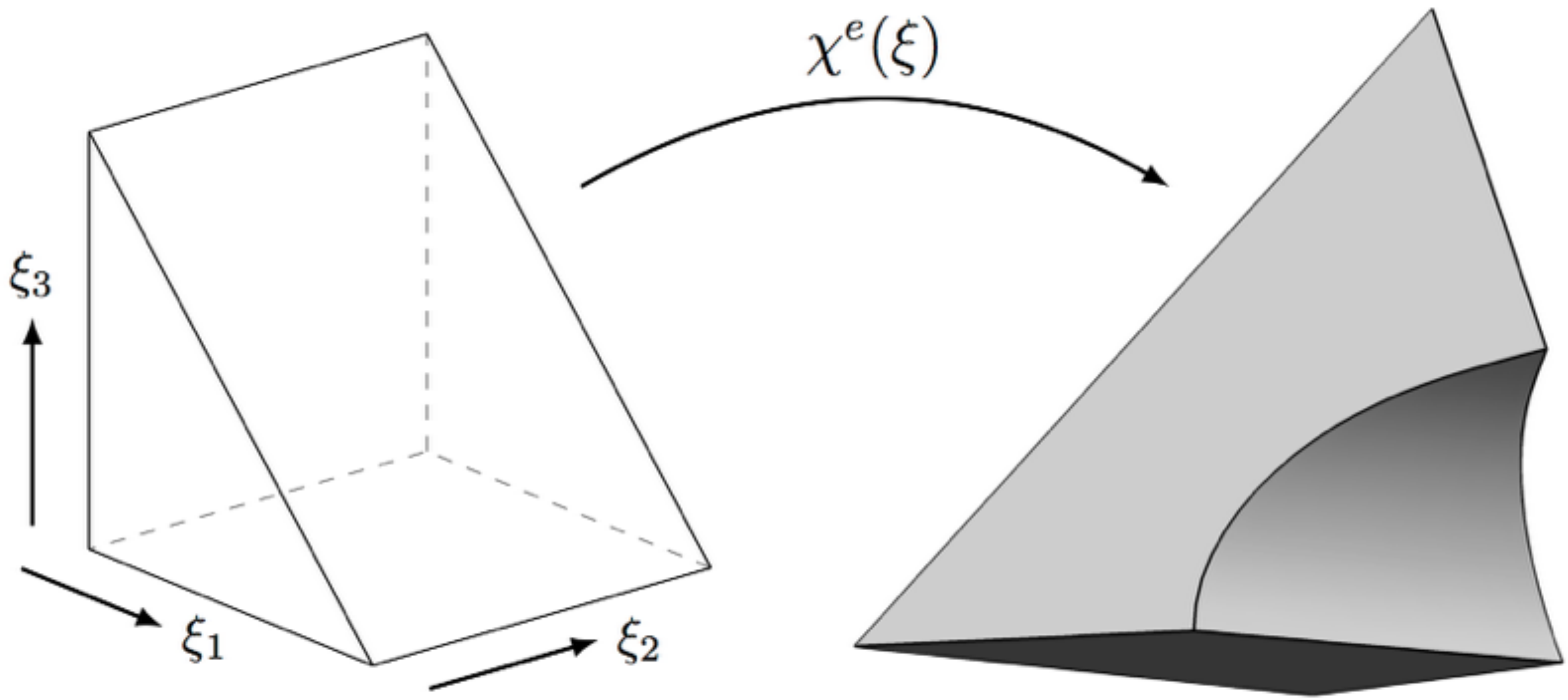
- Initial coarse grid from commercial software
- Apply high-order smoothing technique (e.g. Sherwin & Peiró, 2001) + untangle if necessary
- Refine near walls to produce boundary layer grids

High-order mesh generation

Boundary layer grids are hard to generate:

- High shear near walls
- First element needs to be of size roughly $O(Re^{-2})$
- Unfeasible to run with this number of elements in the entire domain and across surface of wall
- Therefore highly-stretched elements required
- Also has to be coarse for high-order to make sense

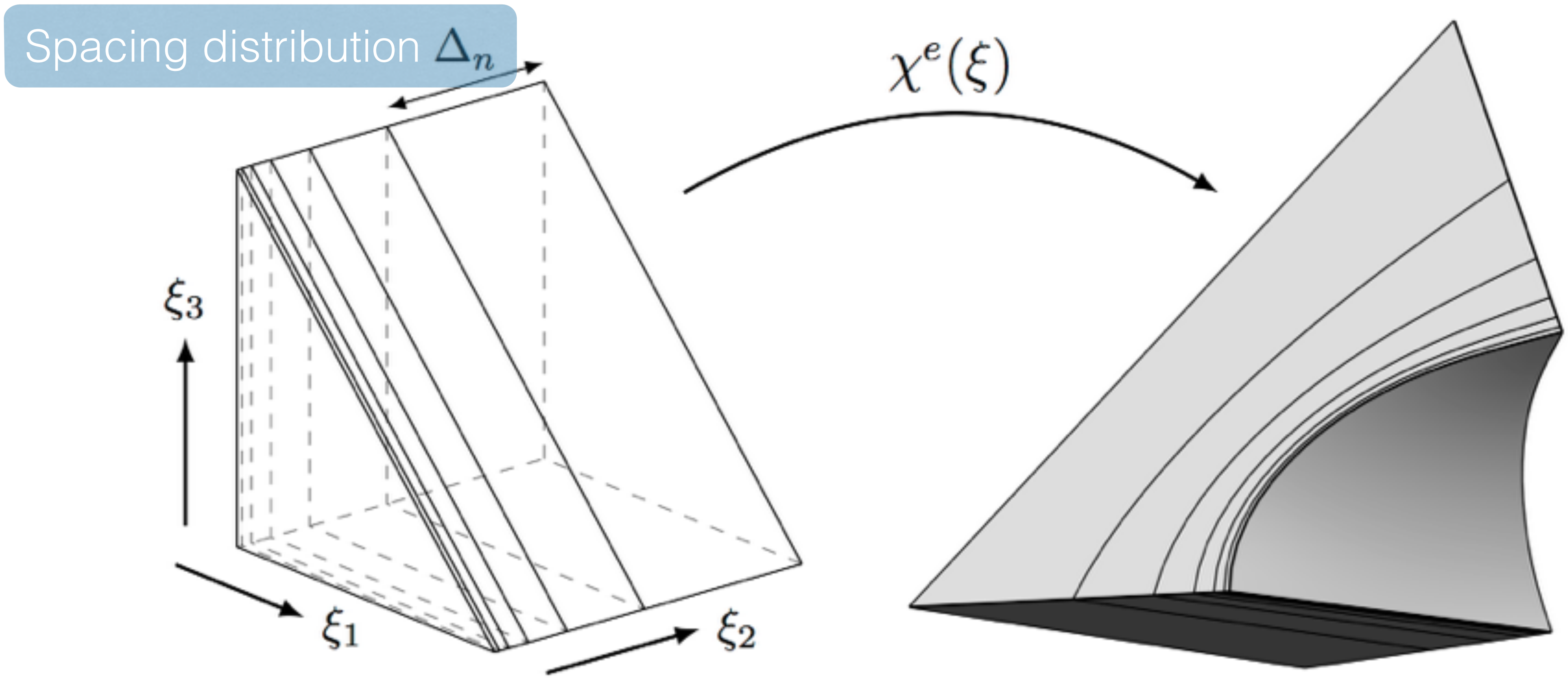
Isoparametric mapping



Shape function is a mapping from reference element (parametric coordinates) to mesh element (physical coordinates)

An isoparametric approach to high-order curvilinear boundary-layer meshing
D. Moxey, M. Hazan, S. J. Sherwin, J. Peiró, under review in Comp. Meth. Appl. Mech. Eng.

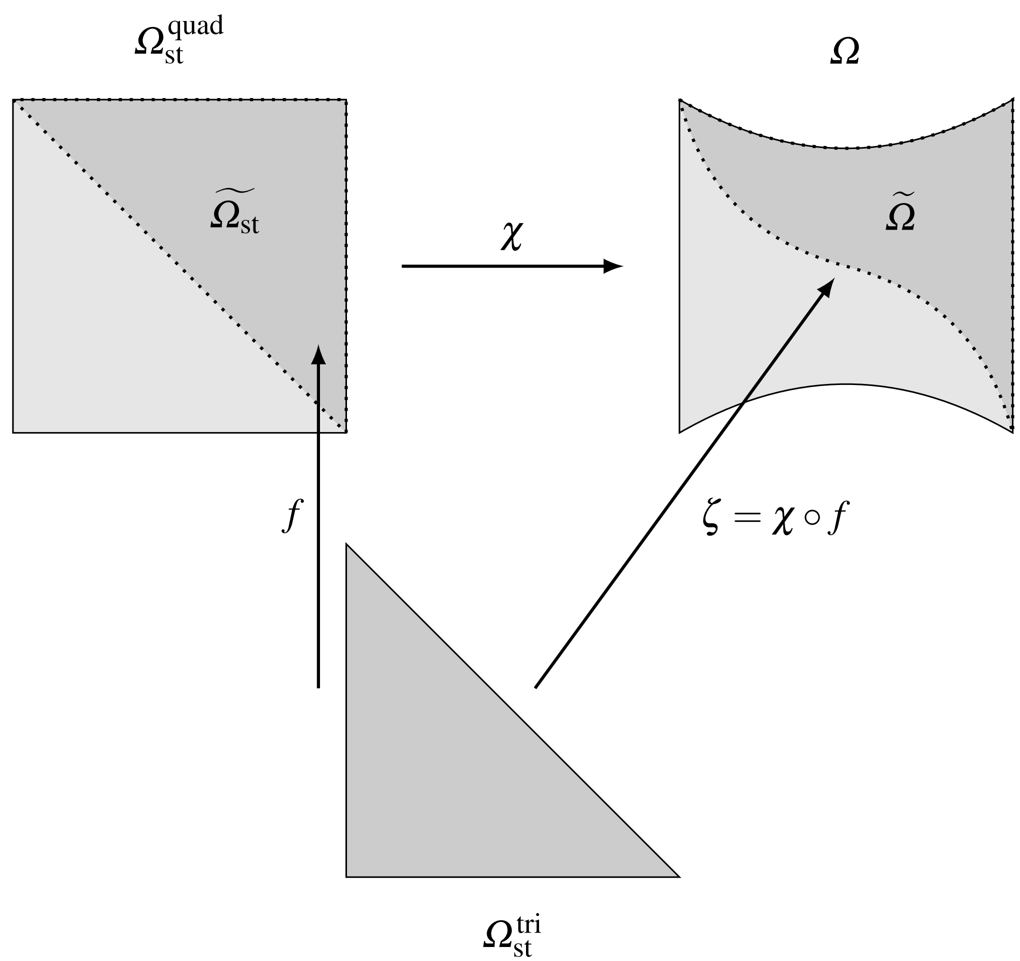
Boundary layer mesh generation



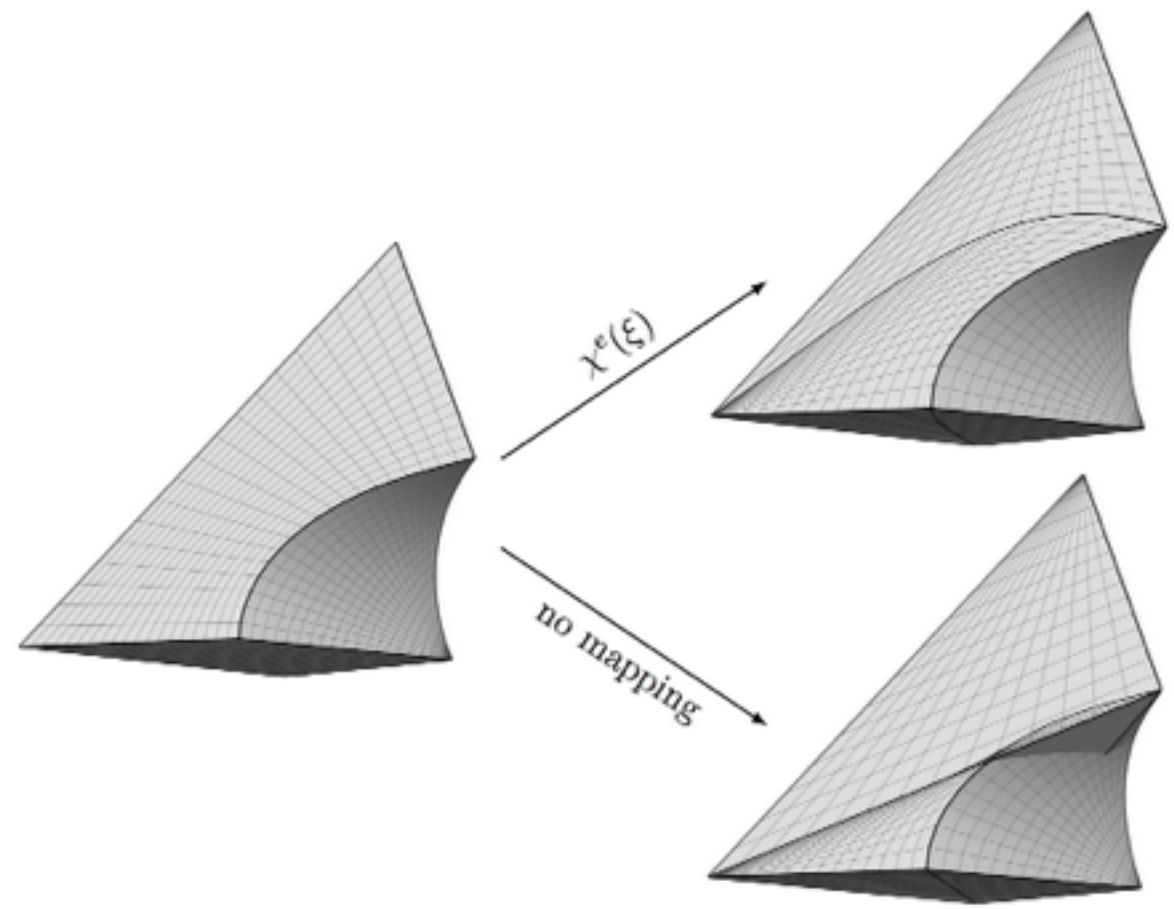
Subdivide the reference element in order to obtain a boundary layer mesh

An isoparametric approach to high-order curvilinear boundary-layer meshing
D. Moxey, M. Hazan, S. J. Sherwin, J. Peiró, under review in Comp. Meth. Appl. Mech. Eng.

More complex transforms



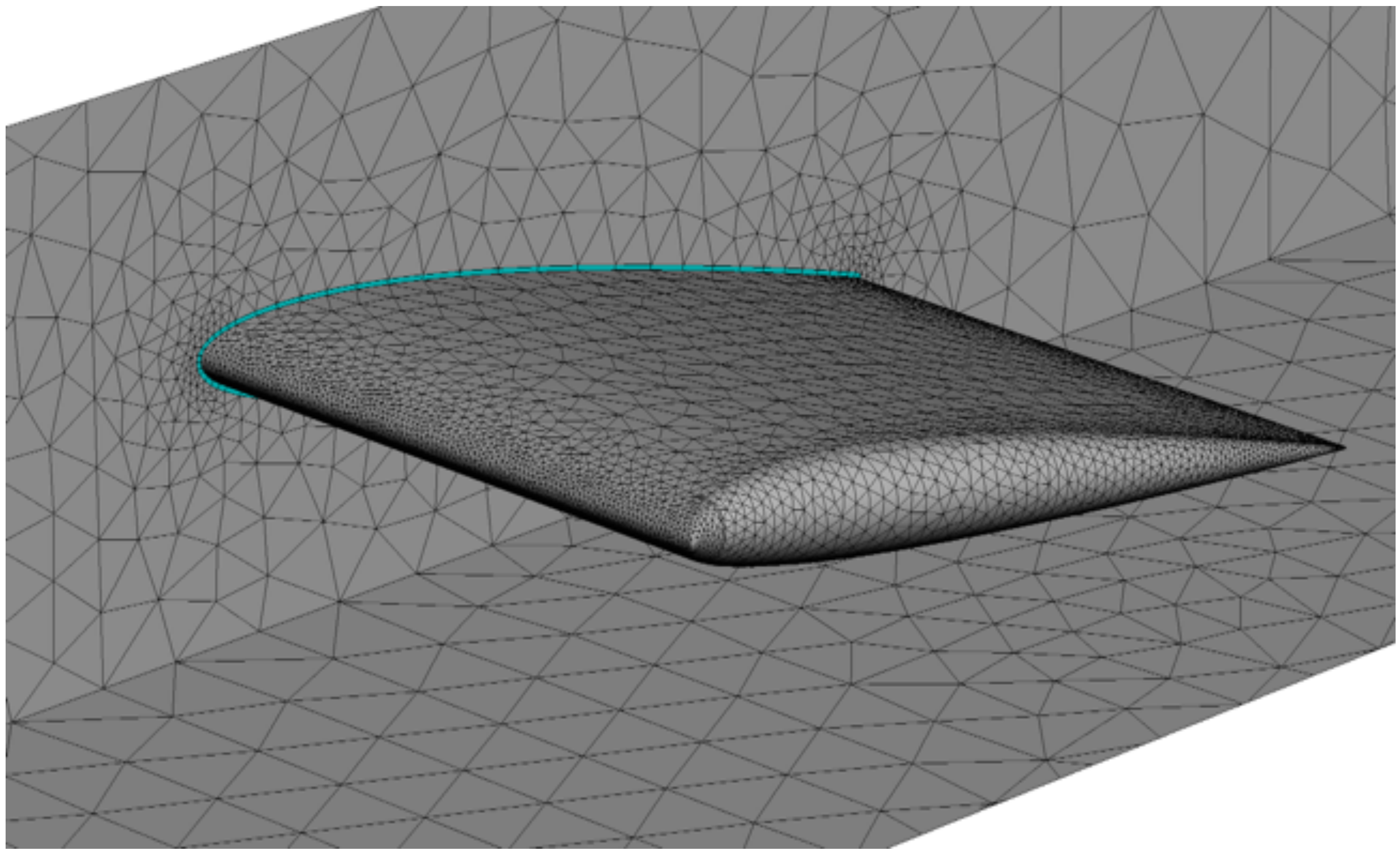
Quads to triangles



Prisms to tetrahedra

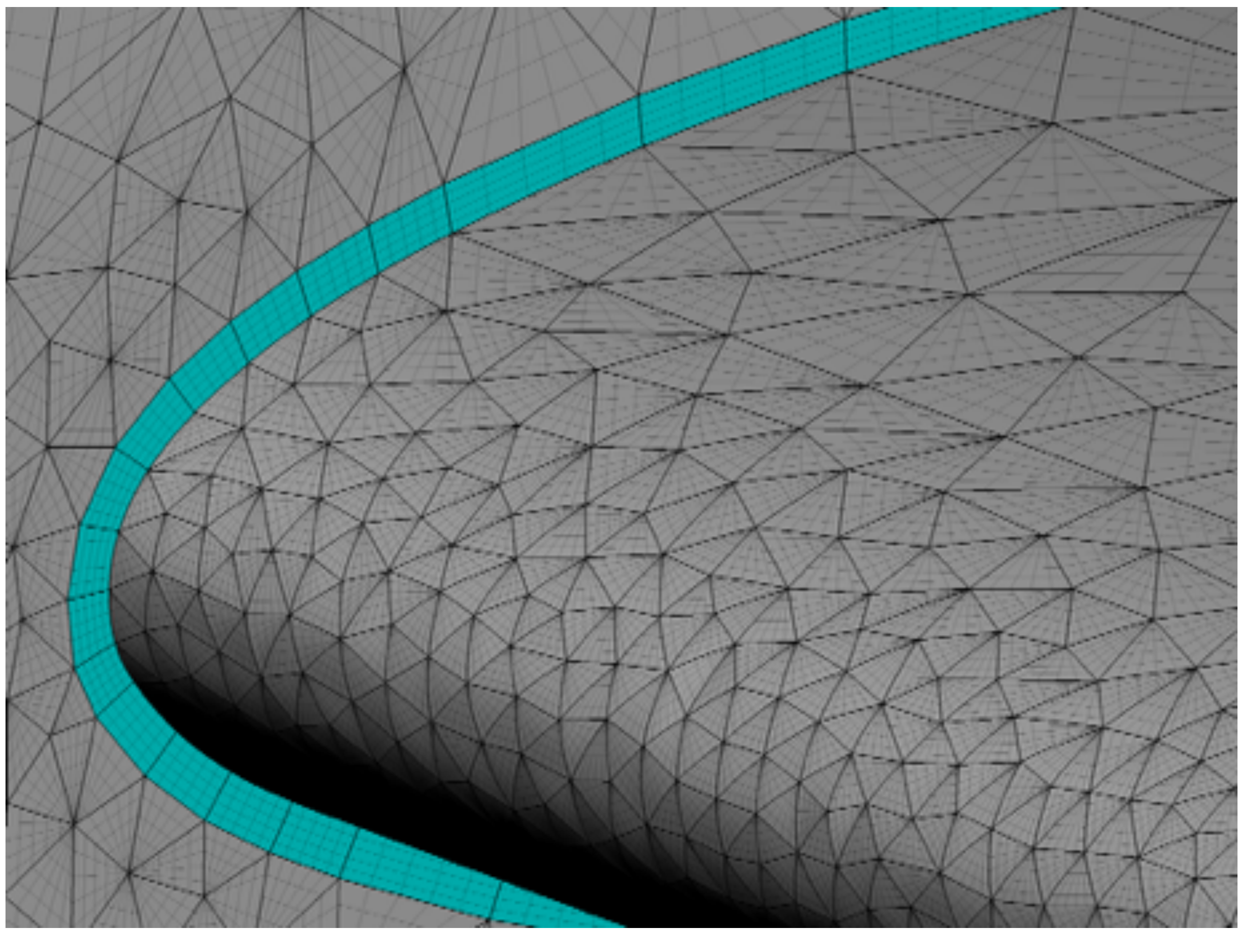
On the generation of curvilinear meshes through subdivision of isoparametric elements
D. Moxey, M. Hazan, S. J. Sherwin, J. Peiró, to appear in proceedings of Tetrahedron IV

NACA 0012 wing case

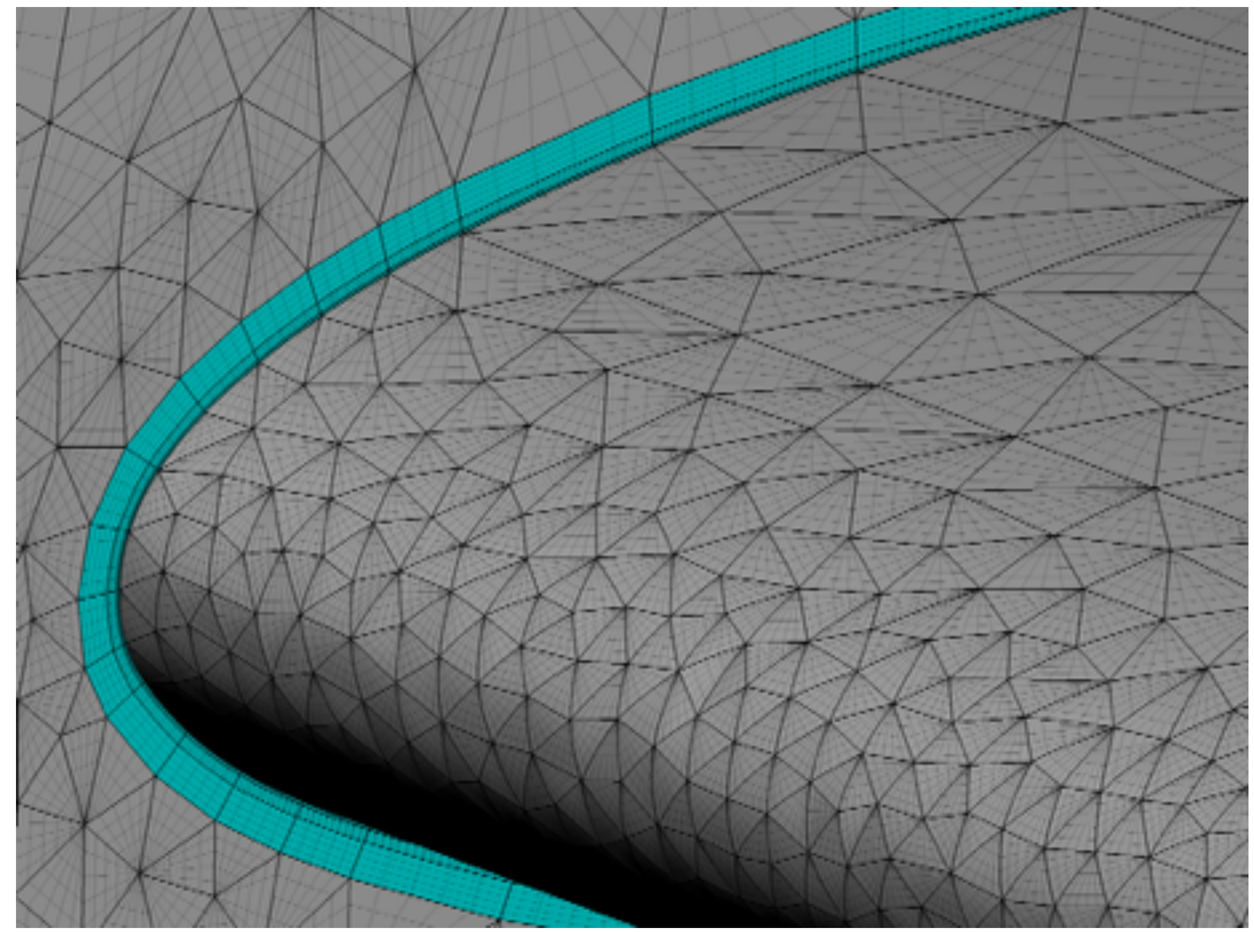


Experimental data available at Re = 4.5m (Chow et al, 1997)

NACA 0012 boundary layer grid



High order mesh
$P = 5$

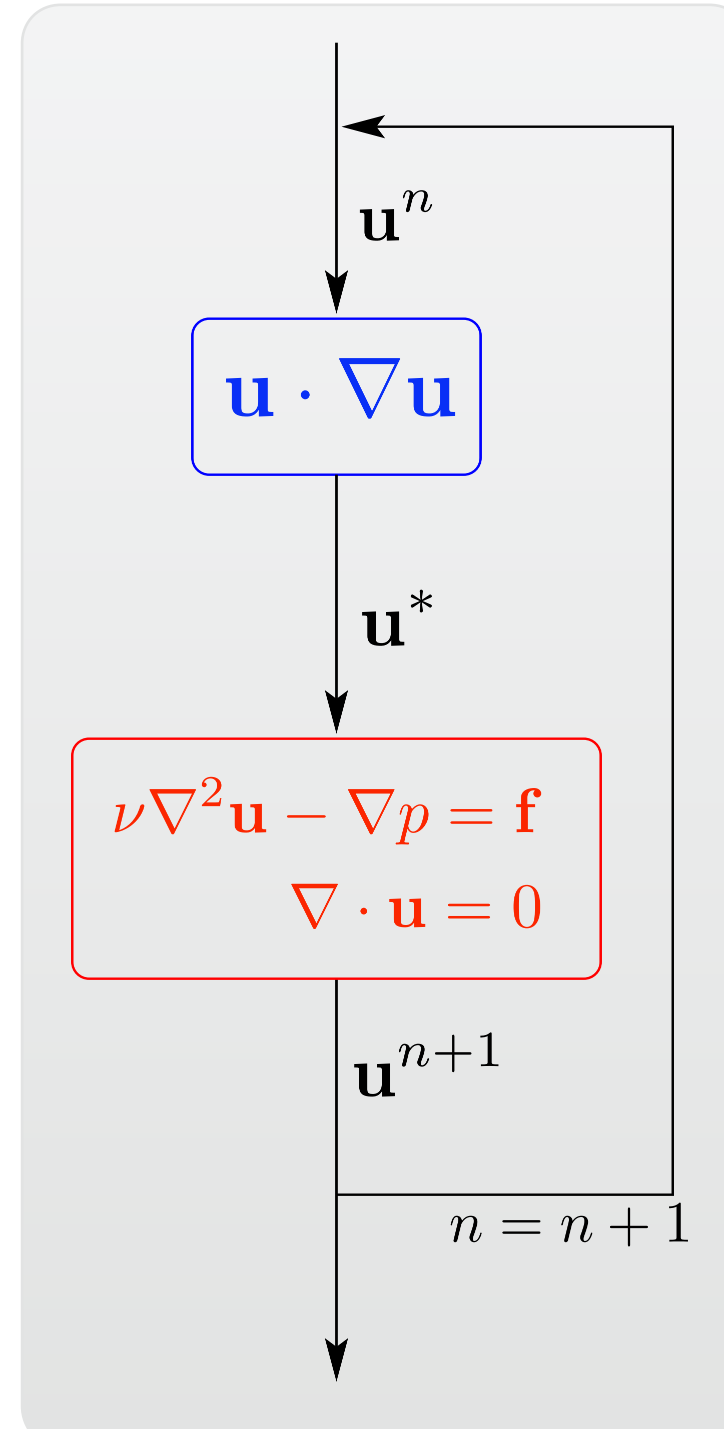


Apply splitting technique

Navier-Stokes Solver

Navier–Stokes:

$$\partial_t \mathbf{u} + \mathbf{N}(\mathbf{u}) = -\nabla p + \nu \nabla^2 \mathbf{u}$$
$$\nabla \cdot \mathbf{u} = 0$$



Velocity correction scheme (aka stiffly stable):

Orszag, Israeli, Deville (90), Karnaidakis Israeli, Orszag (1991), Guermond & Shen (2003)

Advection: $\mathbf{u}^\star = -\sum_{q=1}^{J} \alpha_q \mathbf{u}^{n-q} - \Delta t \sum_{q=0}^{J-1} \beta_q \mathbf{N}(\mathbf{u}^{n-q})$

Pressure Poisson: $\nabla^2 p^{n+1} = \frac{1}{\Delta t} \nabla \cdot \mathbf{u}^*$

Helmholtz: $\nabla^2 \mathbf{u}^{n+1} - \frac{\alpha_0}{\nu \Delta t} \mathbf{u}^{n+1} = -\frac{\mathbf{u}^\star}{\nabla \Delta t} + \frac{1}{\nu} \nabla p^{n+1}$

Challenge: Stabilisation

Instability arises through (at least) two routes:

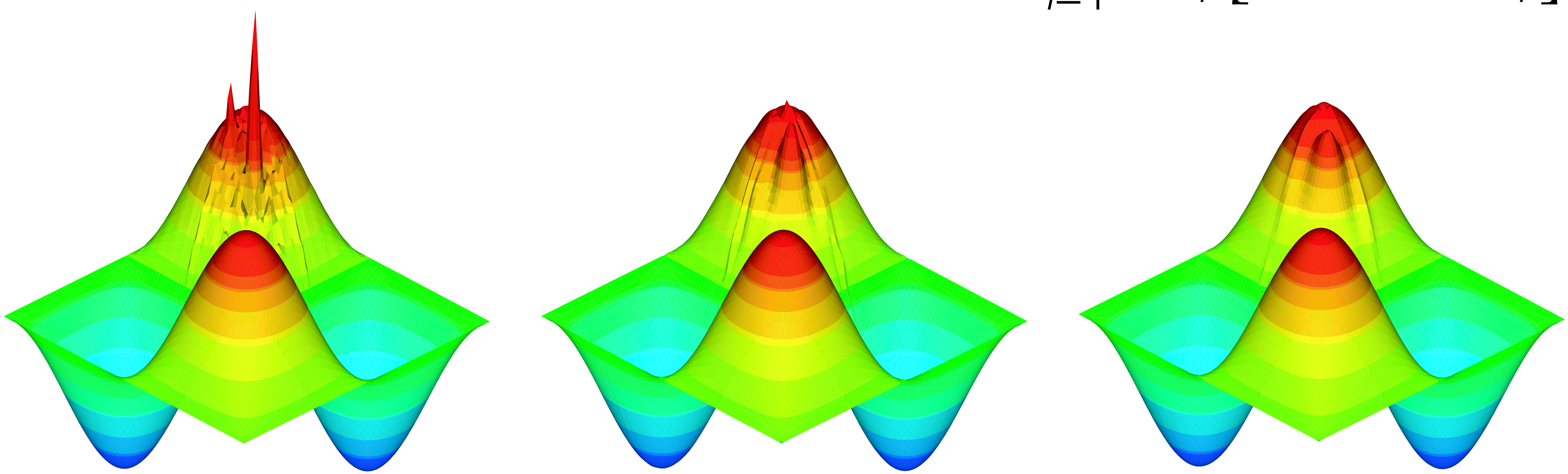
- Consistent integration of nonlinear terms
- Insufficient dissipation from the numerical method

Here we use

- Over-integration of nonlinear terms
- Spectral vanishing viscosity

Spectral Vanishing Viscosity

$$\frac{\partial u}{\partial t} = \nu \nabla^2 u + S_{VV}(u), \qquad S_{VV}(u) = \varepsilon \sum_{i=1}^{\dim} \frac{\partial}{\partial x_i}\left[Q_{\dim} \star \frac{\partial u}{\partial x_i} \right]$$



No SVV

$P_{cut} = 7,$
$\epsilon_{SVV} = 0.1$

$P_{cut} = 3,$
$\epsilon_{SVV} = 0.1$

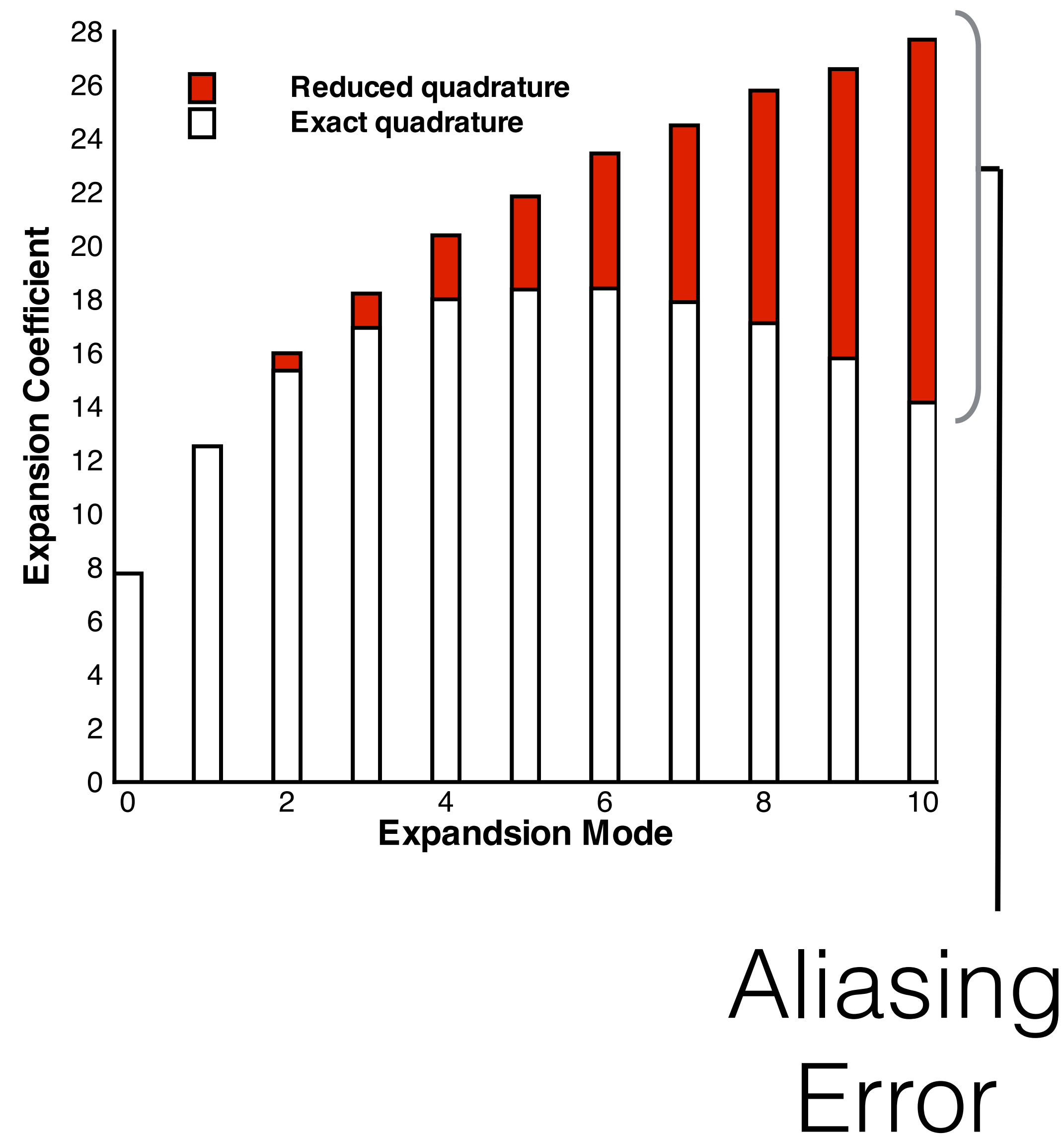
R. Kirby, S. Sherwin, *Comp. Meth. Appl. Mech. Eng.*, 2006

Aliasing

Example: $u(\xi) = \sum_{k=0}^{10} u_k \psi_i(\xi)$

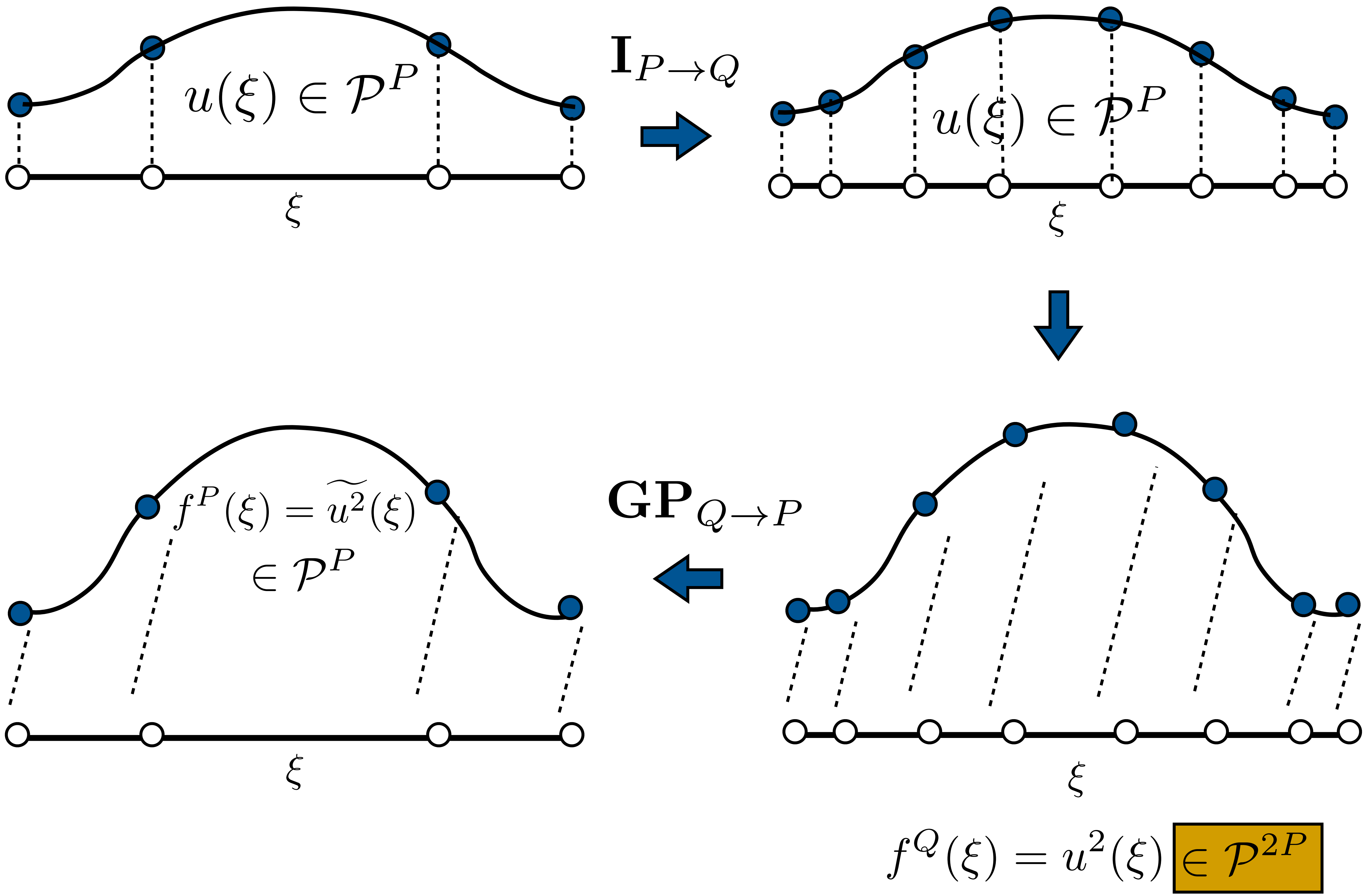
Galerkin projection of u^2 using:

- Q = 17 – exact Quadrature
- Q = 12 – sufficient for integrating 20th degree polynomials

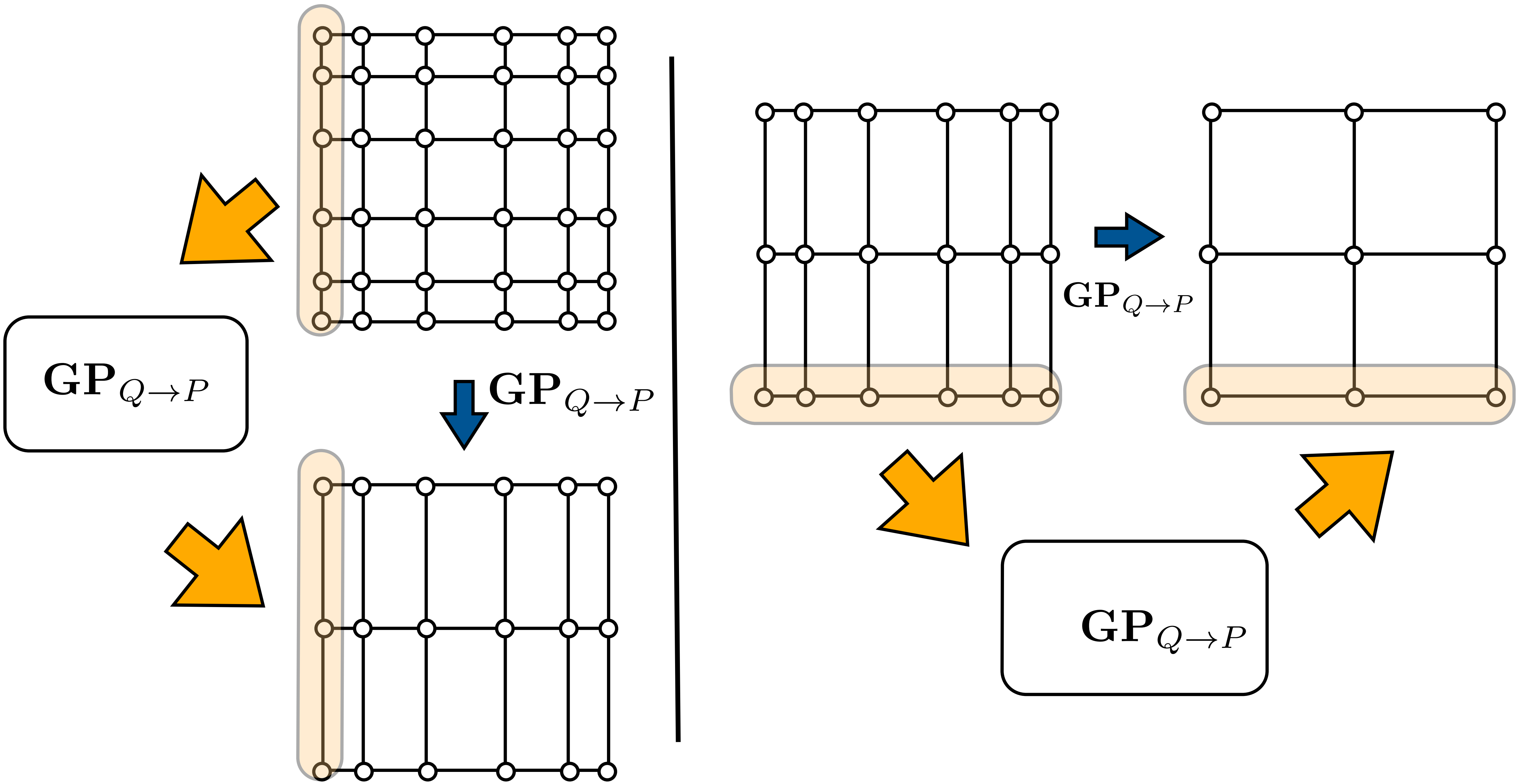


Example from Kirby & Karniadakis, J. Comp. Phys (2003)

Overview of nodal projection of u^2



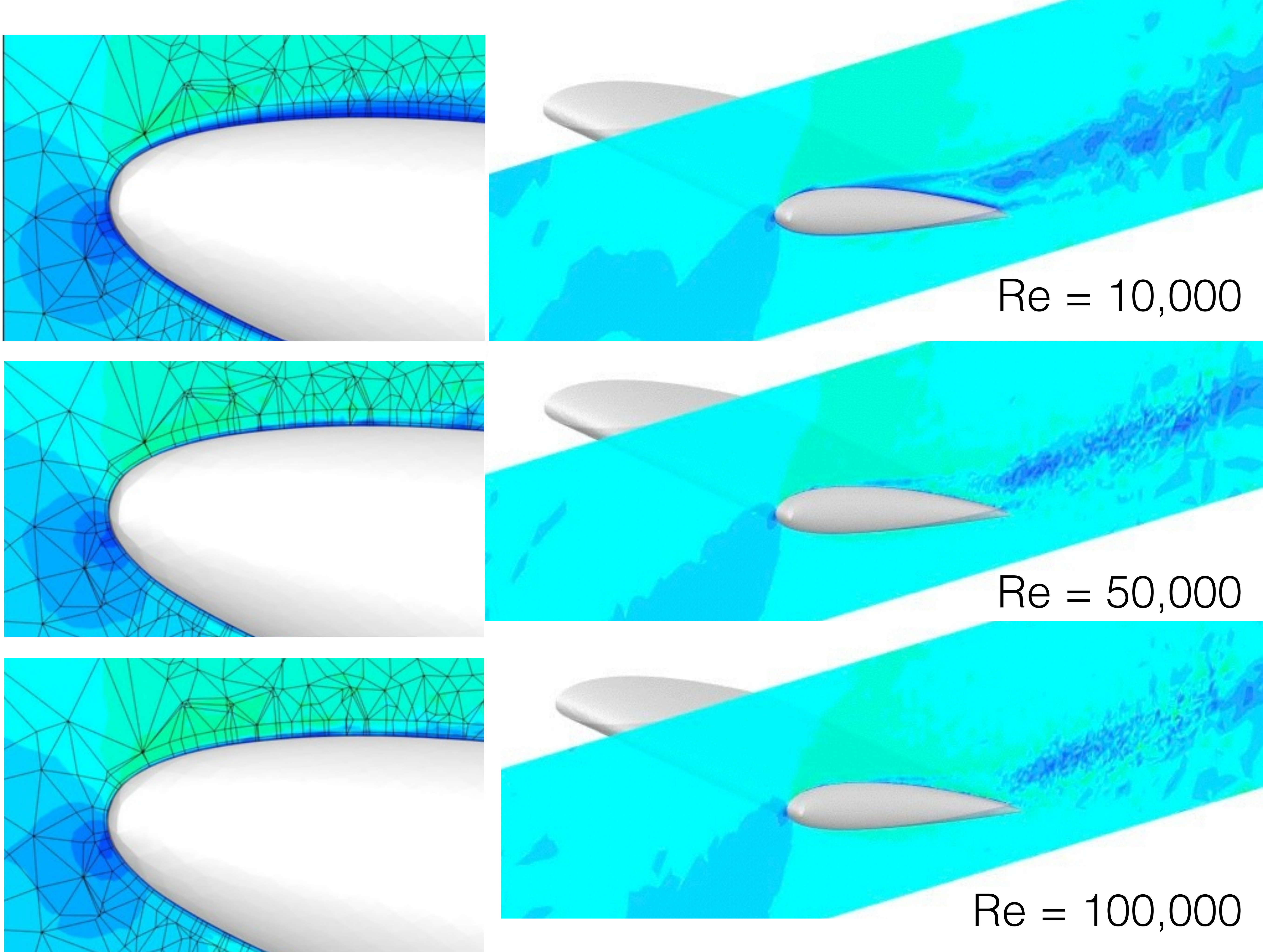
Use tensor product structure



$$P \times Q^2 + P^2 \times Q \Rightarrow O(P^3)$$

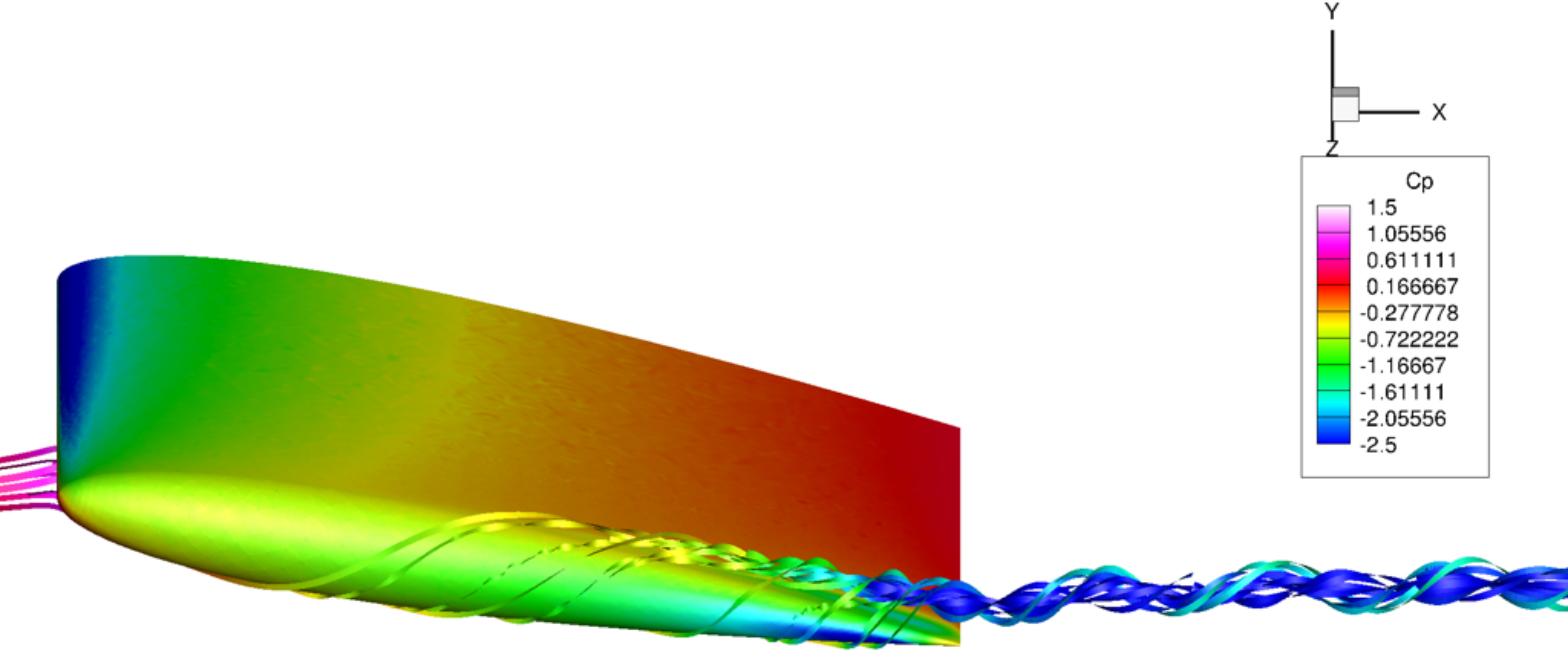
Essentially performing sum factorisation

In 3D: $O(P^6)$ vs. $O(P^4)$



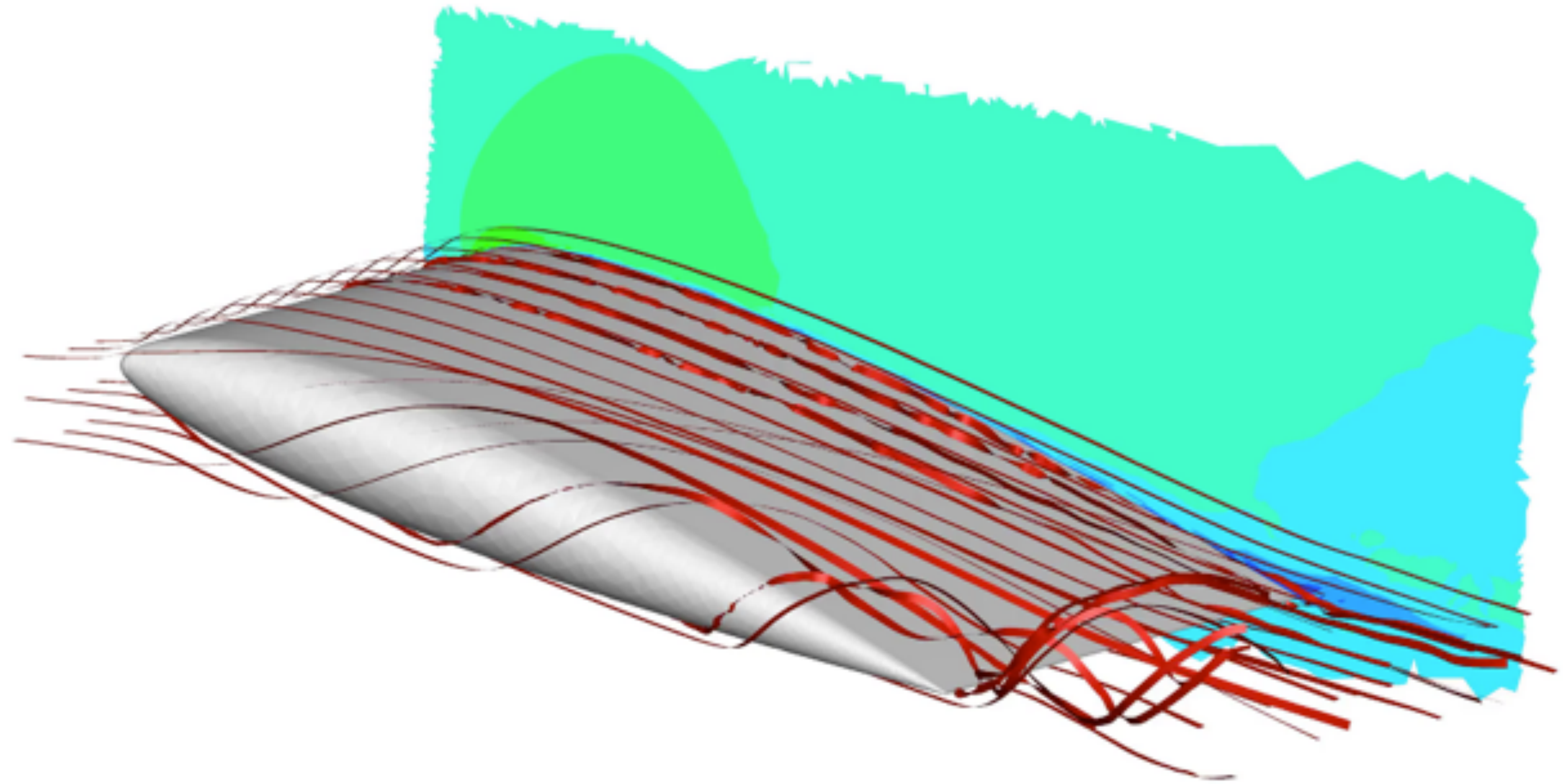
Re = 10,000
Re = 50,000
Re = 100,000

Flow characteristics

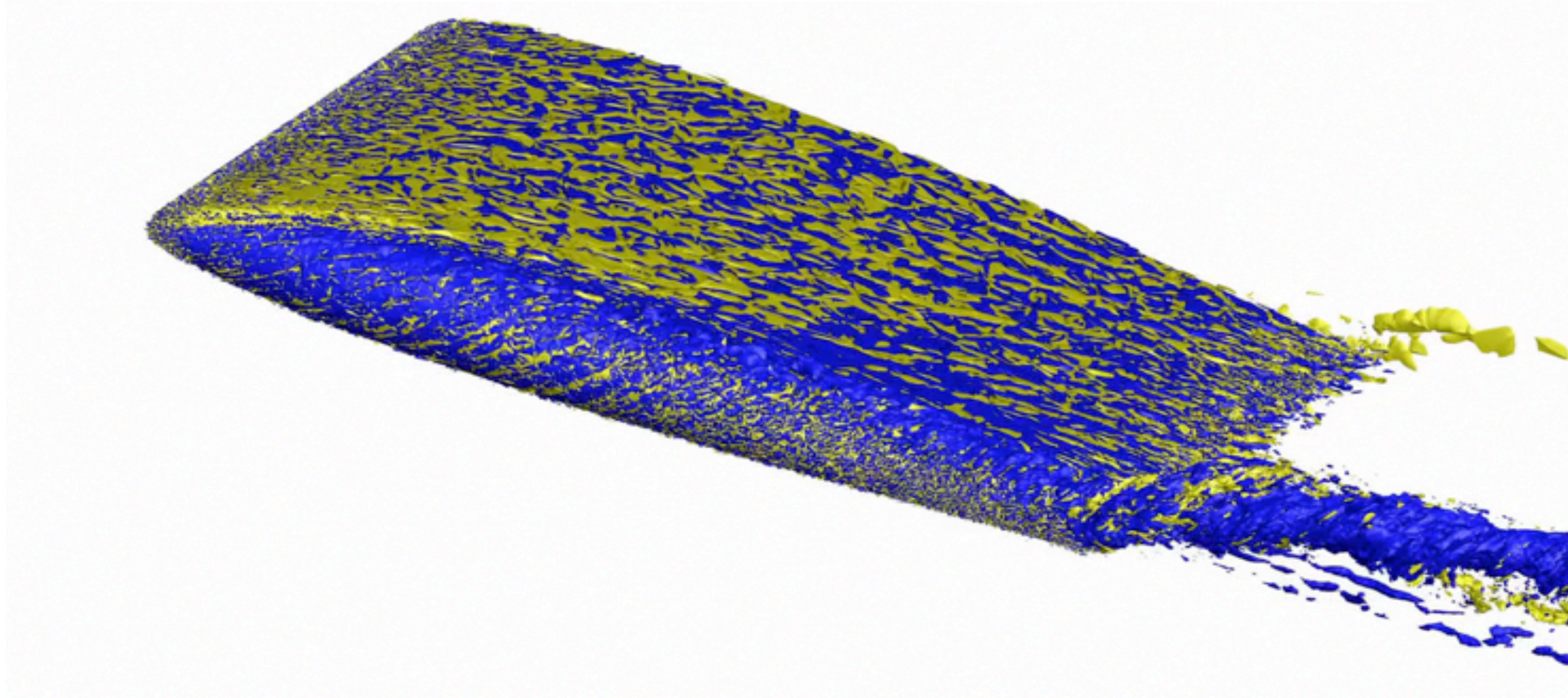


NACA 0012 wing tip (Re = 1.2M)

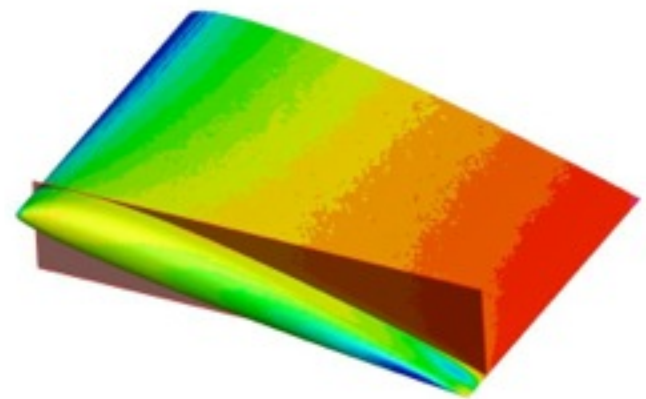
Streamlines

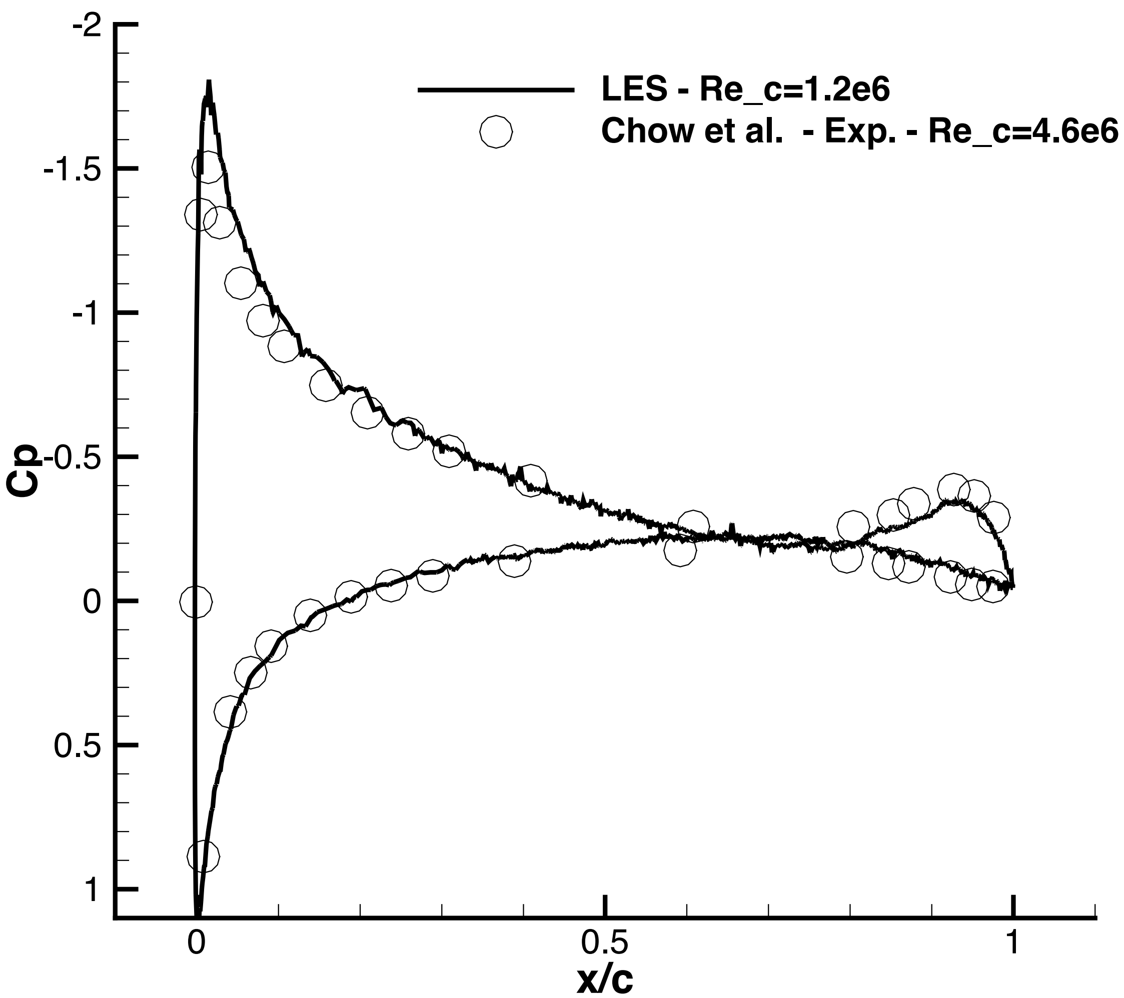


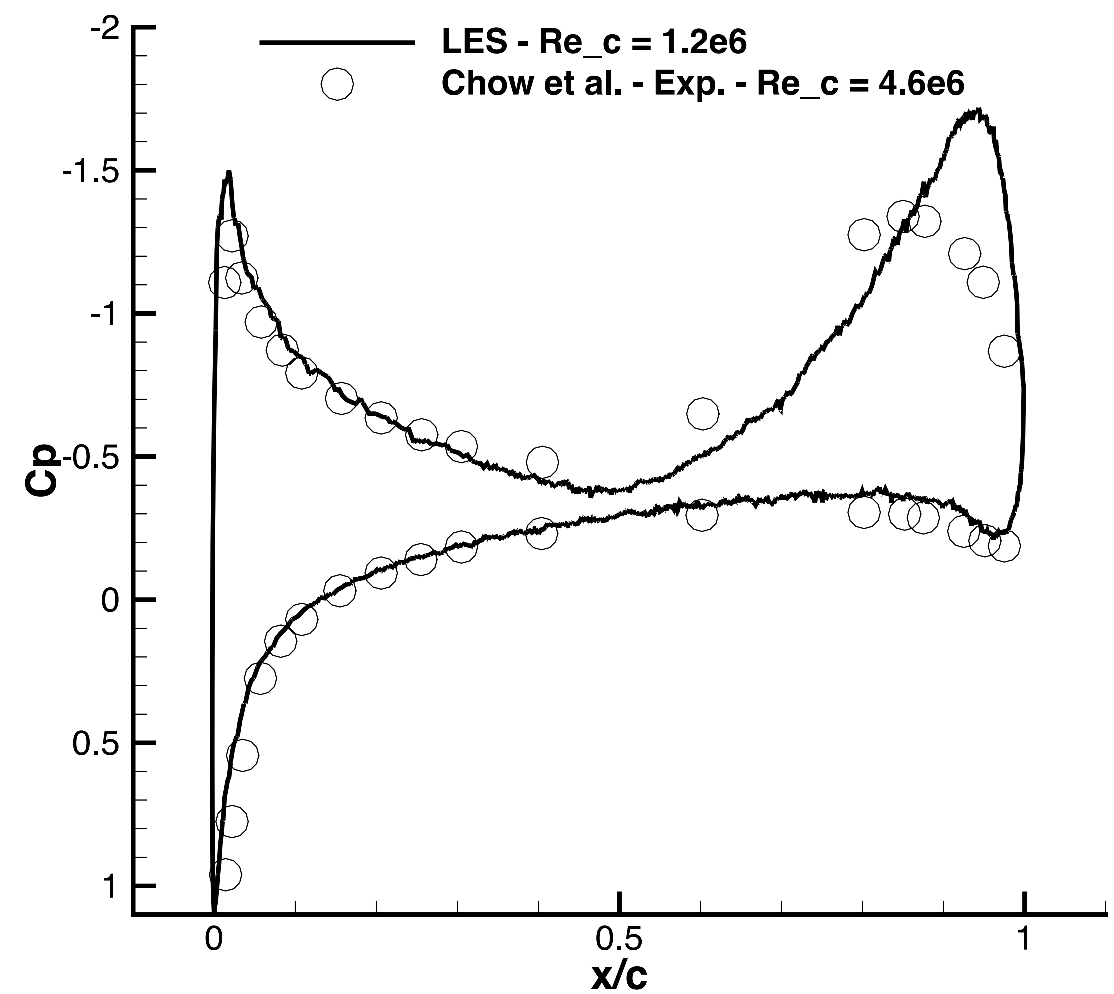
Streamwise vorticity



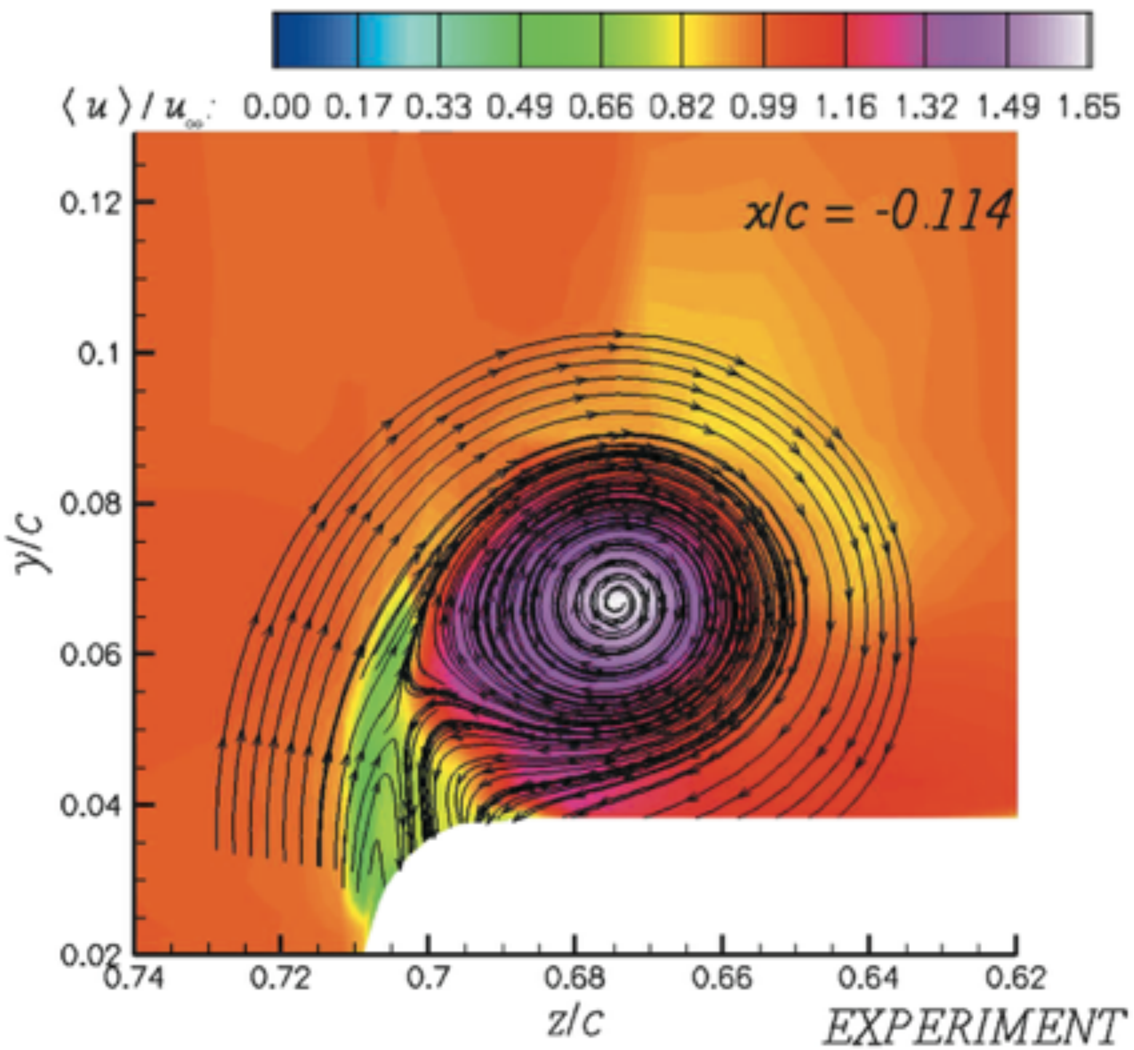
Pressure coefficient distribution



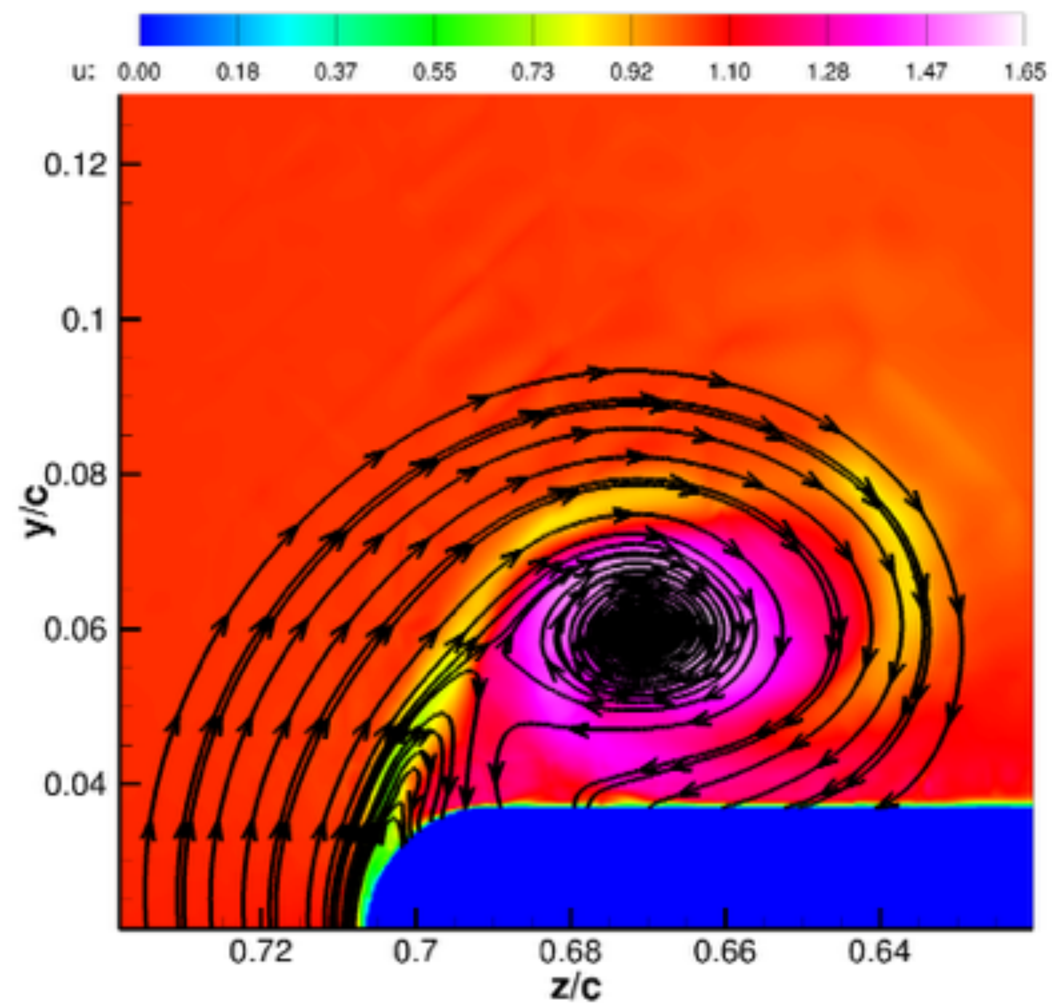




Vortex core



Experiment



Simulation

Conclusions

- High-order methods can be applied to these problems and successfully capture essential flow dynamics

- Still a need for high-order mesh generation strategies for coarse grid

- Promising results for larger and more complex geometries

Thanks for listening!