Applications of the spectral/hp element method to CFD

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Overview

- Motivation
- Some challenges
- A few results

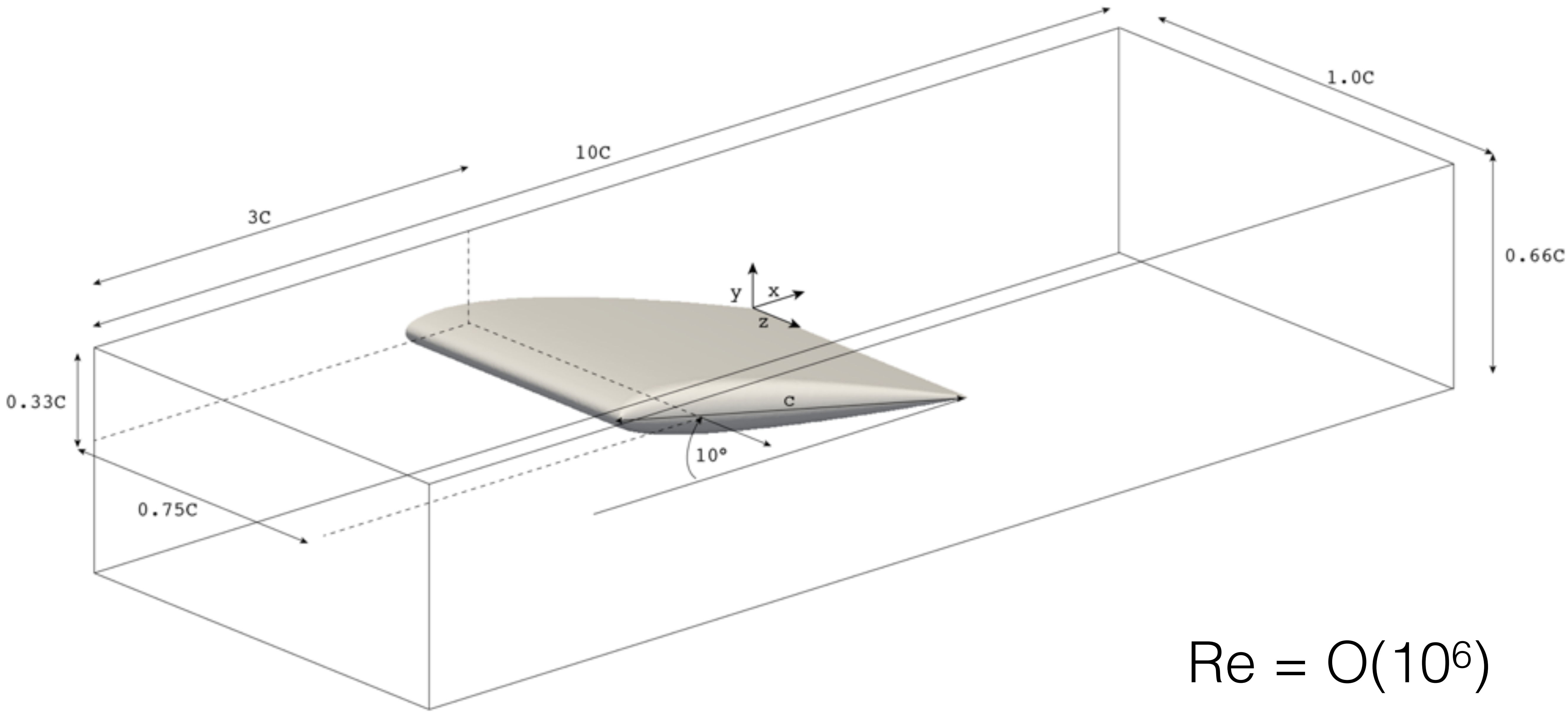
Motivation

Primary research goal is to investigate challenging external aerodynamics cases:

- High Reynolds numbers
- Complex three-dimensional geometries
- Large resolution requirements
- Transient dynamics

Using DNS + stabilisation

NACA 0012 wing tip



Re = O(10^6)

Difficult to capture transient effects with RANS

Nektar++ high-order framework

Framework for spectral(/hp) element method:

- Dimension independent, supports CG/DG/HDG
- Mixed elements (quads/tris, hexes, prisms, tets, pyramids)
- Solvers for (in)compressible Navier-Stokes, advection-diffusion-reaction, ...
- Parallelised with MPI, scales up to ~10k cores

http://www.nektar.info/

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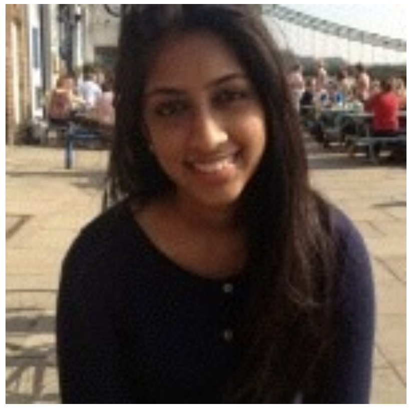

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Challenge(s) for this application

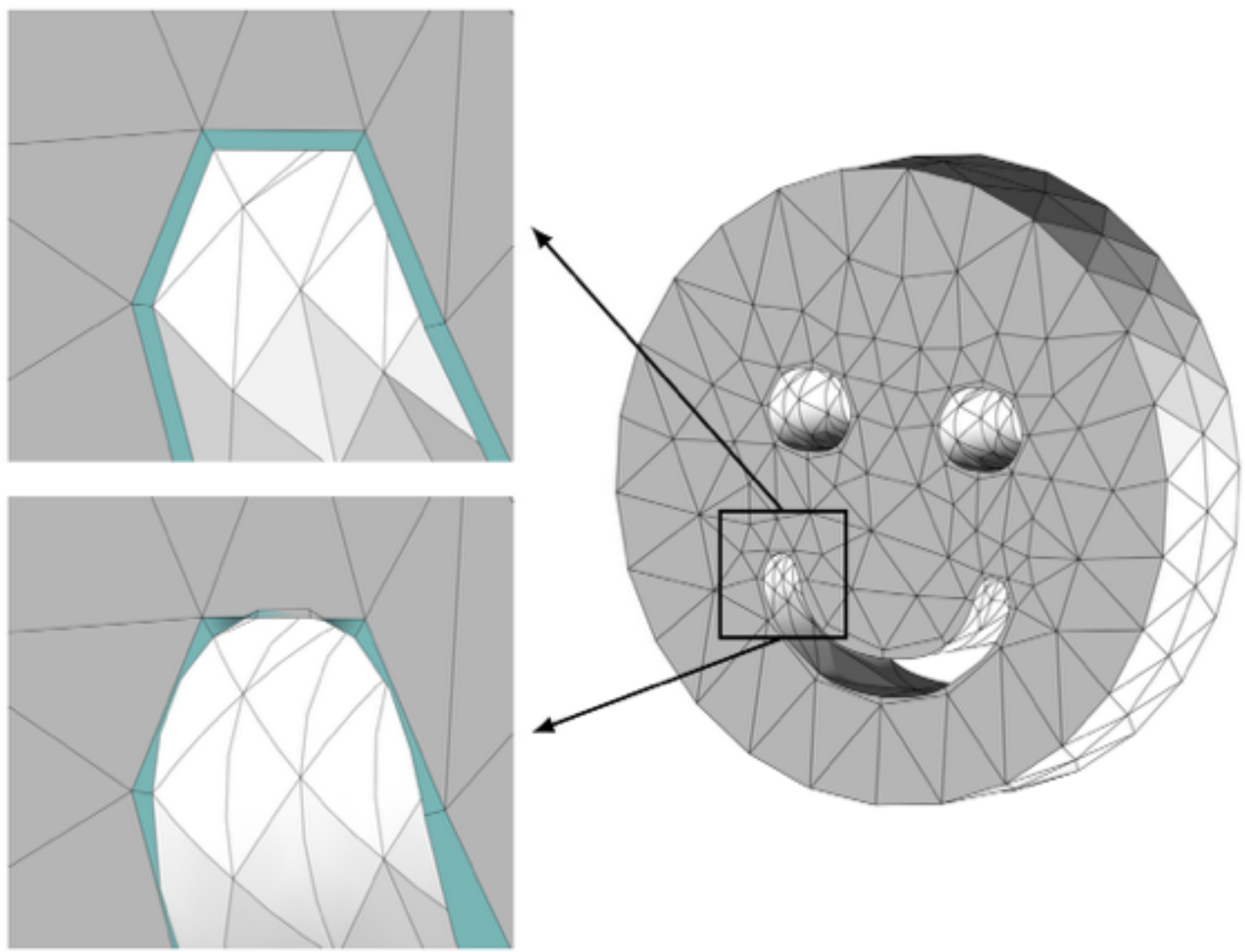
- Stabilisation for high Reynolds number
- Mesh generation

Focus on mesh generation:

- No self-intersecting elements
- Curvilinear elements aligning with geometry
- Deal with boundary layers

High-order mesh generation

Curving mesh often leads to invalid elements



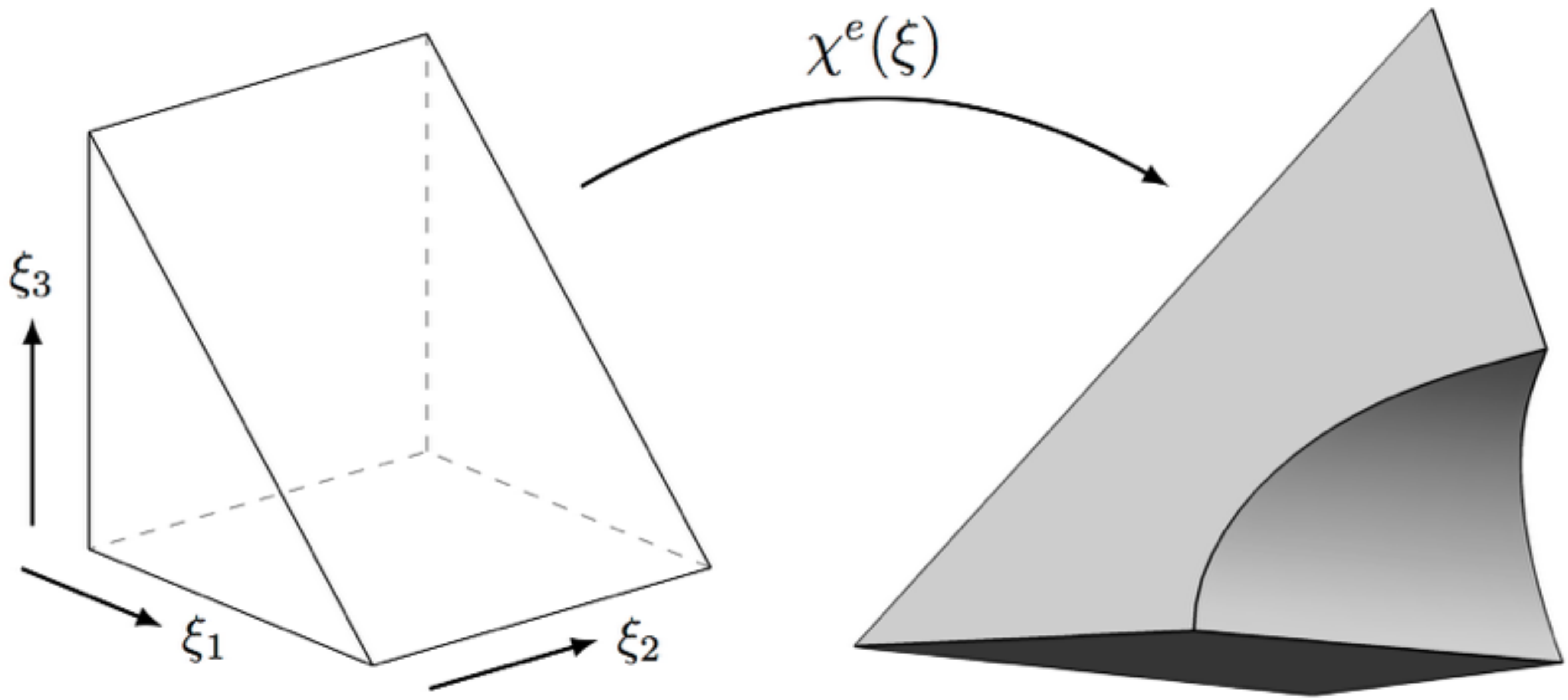
High-order mesh generation

Boundary layer grids are pretty hard to generate:

- High shear near walls
- First element needs to be of size roughly O(Re^{-2})
- Unfeasible to run with this number of elements in the entire domain and across surface of wall
- Therefore highly-stretched elements required
- Also has to be coarse for high-order to make sense

What if we already have a coarse grid?

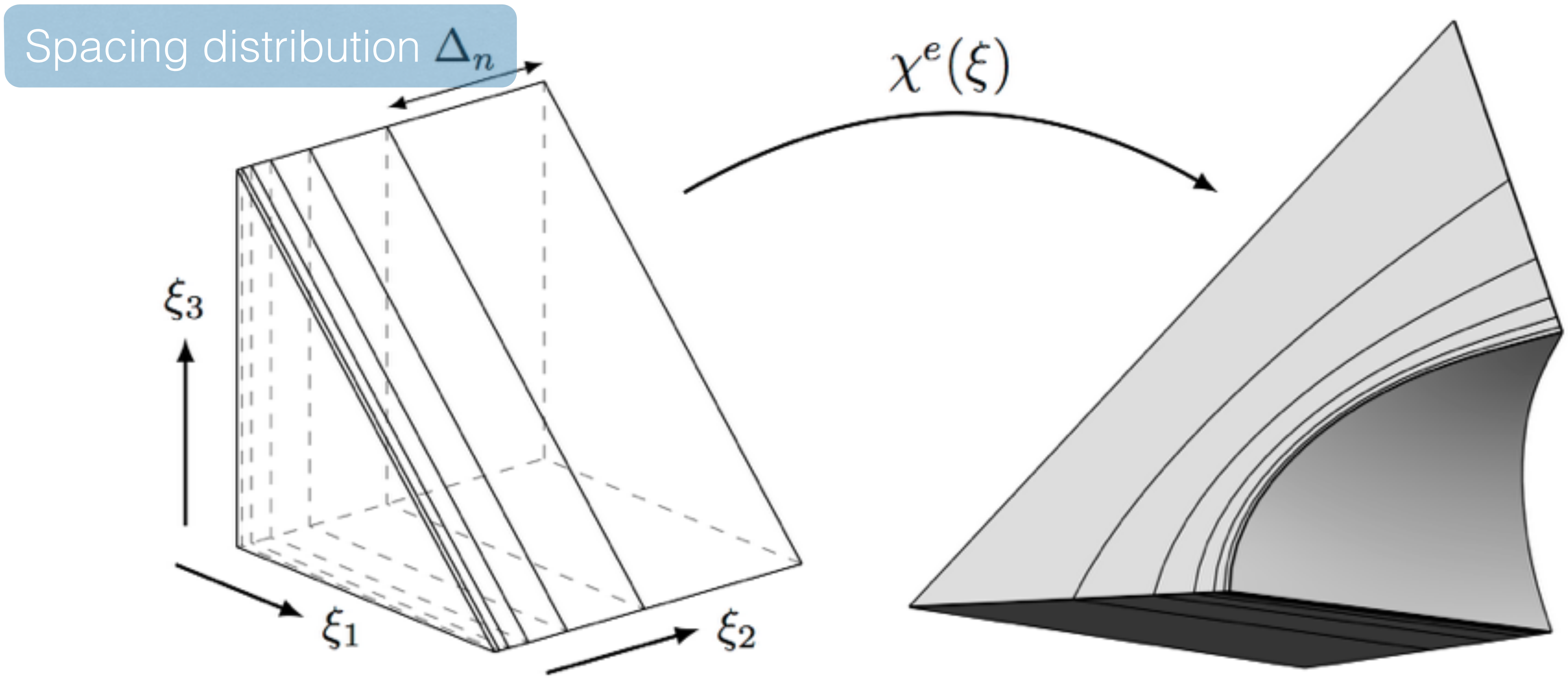
Isoparametric mapping



Shape function is a mapping from reference element (parametric coordinates) to mesh element (physical coordinates)

An isoparametric approach to high-order curvilinear boundary-layer meshing
D. Moxey, M. Hazan, S. J. Sherwin, J. Peiró, under review in Comp. Meth. Appl. Mech. Eng.

Boundary layer mesh generation



Subdivide the reference element in order to obtain a boundary layer mesh

An isoparametric approach to high-order curvilinear boundary-layer meshing
D. Moxey, M. Hazan, S. J. Sherwin, J. Peiró, under review in Comp. Meth. Appl. Mech. Eng.

Some nice properties

- Efficient (no deformation required), relatively easy to implement
- Arbitrarily thin elements can be generated near walls (use geometric progression for spacing)
- Guaranteed to produce valid meshes if original mesh is valid thanks to the chain rule
- For same reason, can calculate Jacobian of subelements *a priori*: quality depends on original coarse grid

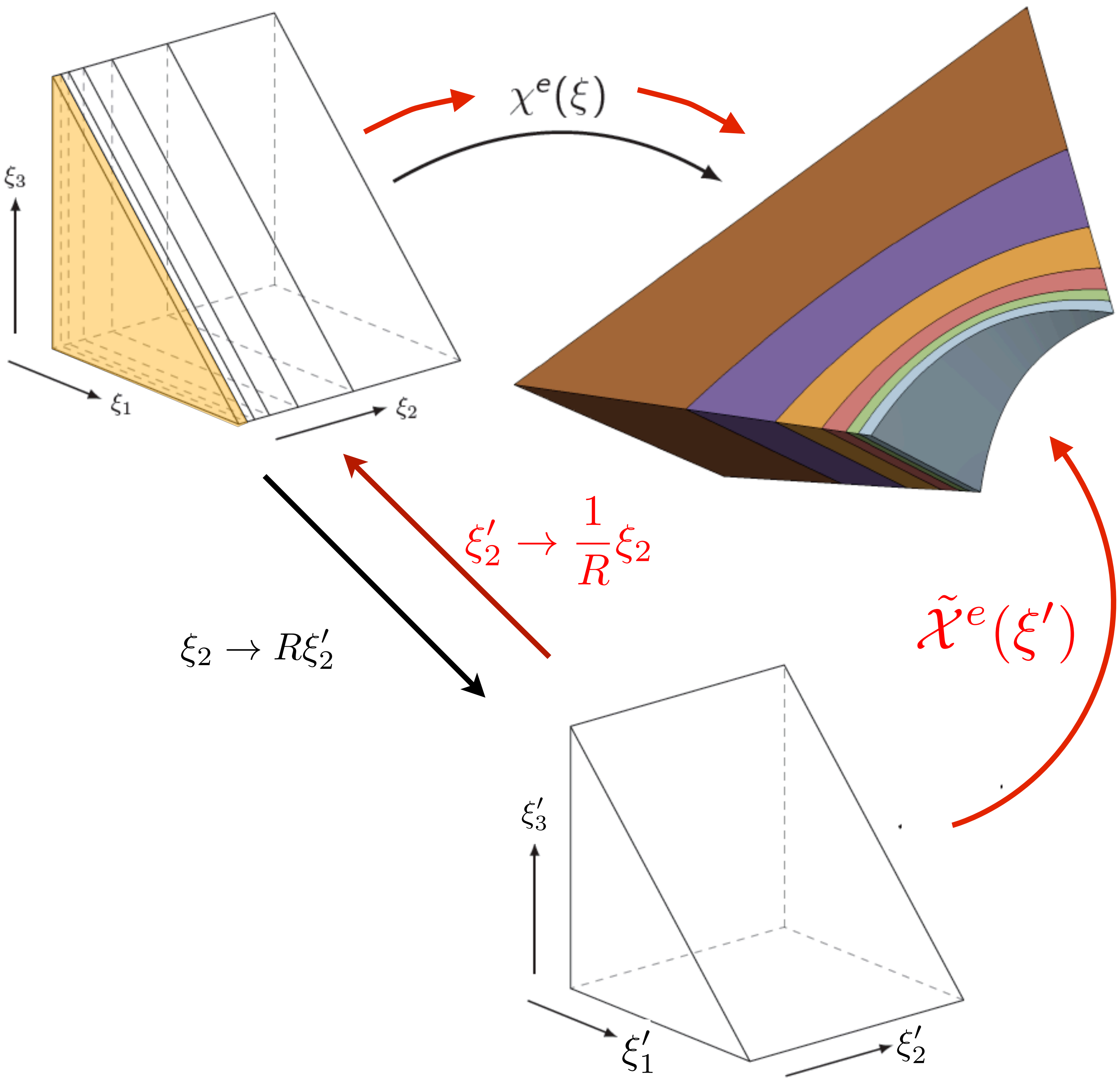
Some drawbacks

- Relies on O-type geometry unless more complex strategies are undertaken

- Relies on validity of coarse grid

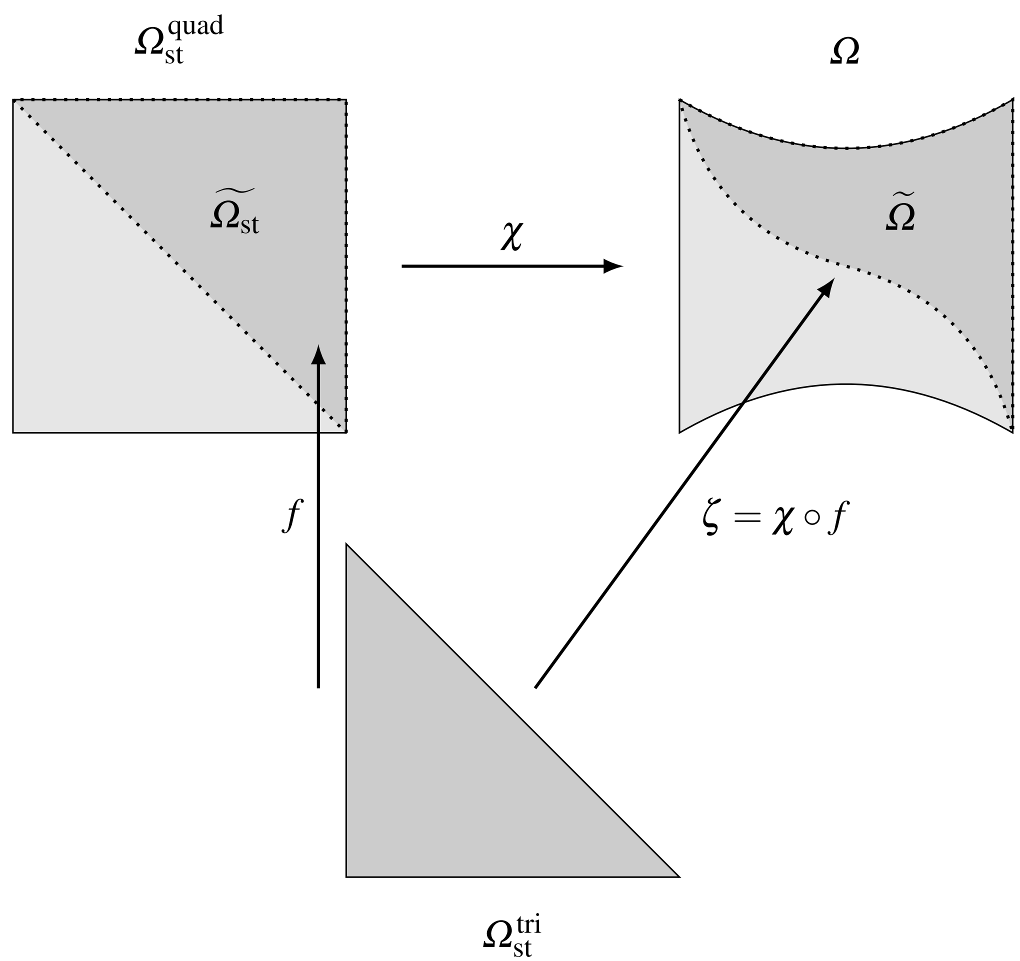
- Mesh quality is dependent on coarse grid

Why does this work?

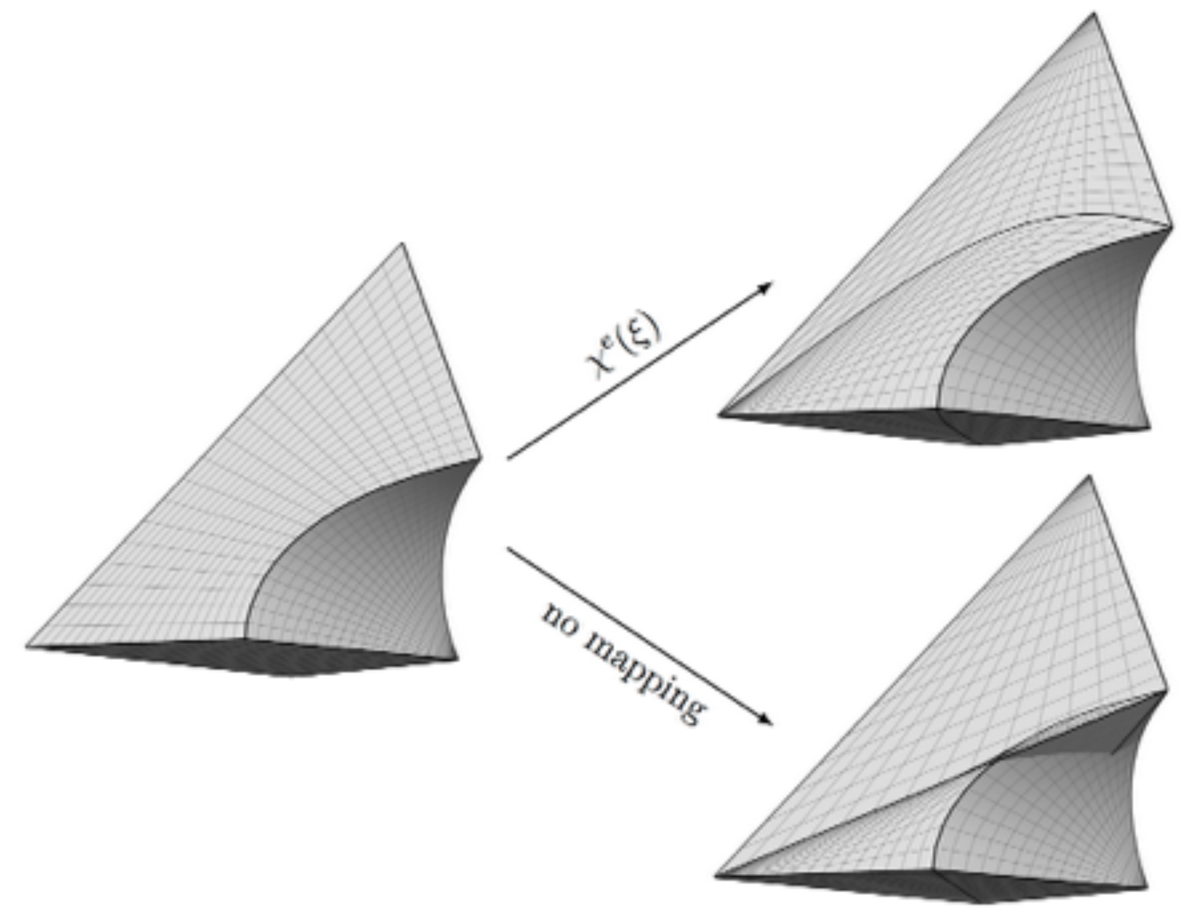


Jacobian of refined prism is a scaled version of Jacobian of original map

More complex transforms



Quads to triangles



Prisms to tetrahedra

On the generation of curvilinear meshes through subdivision of isoparametric elements
D. Moxey, M. Hazan, S. J. Sherwin, J. Peiró, to appear in proceedings of Tetrahedron IV

Why does this work? (1)

- Write mapping in tensor product of modal functions

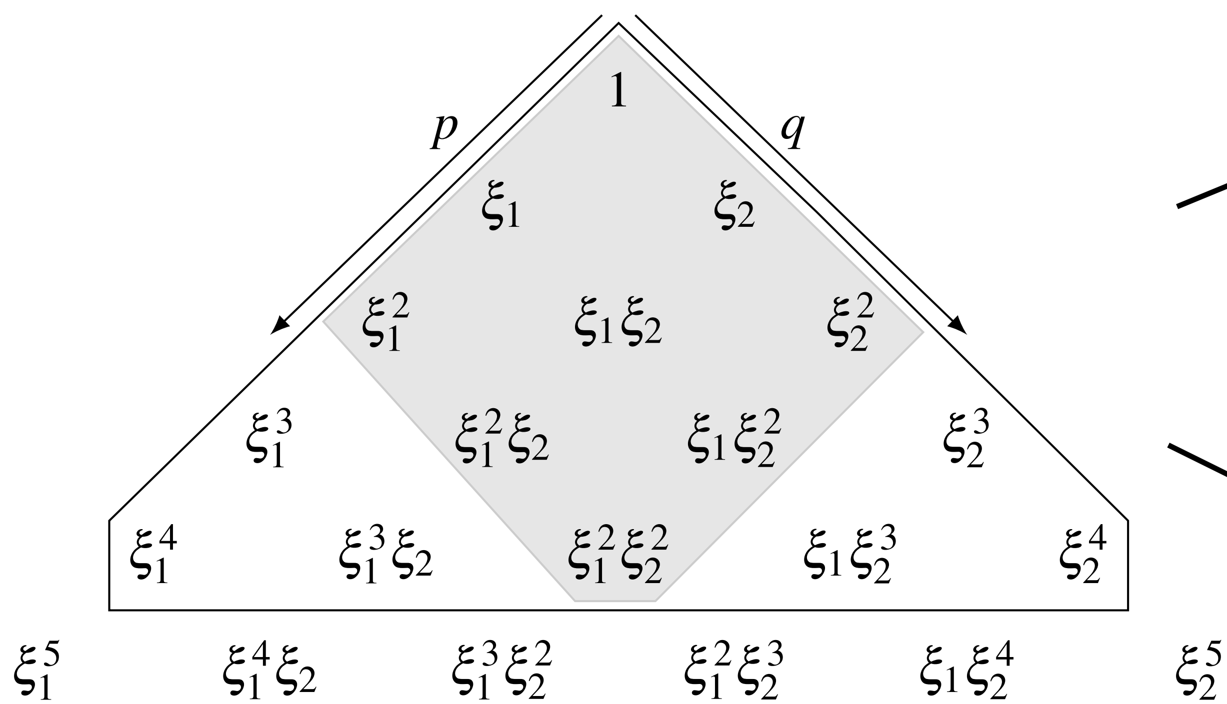
$$\chi_i^e(\xi_1, \xi_2) = \sum_{p,q} (\hat{\chi}_i^e)_{pq} \psi_p(\xi_1) \psi_q(\xi_2)$$

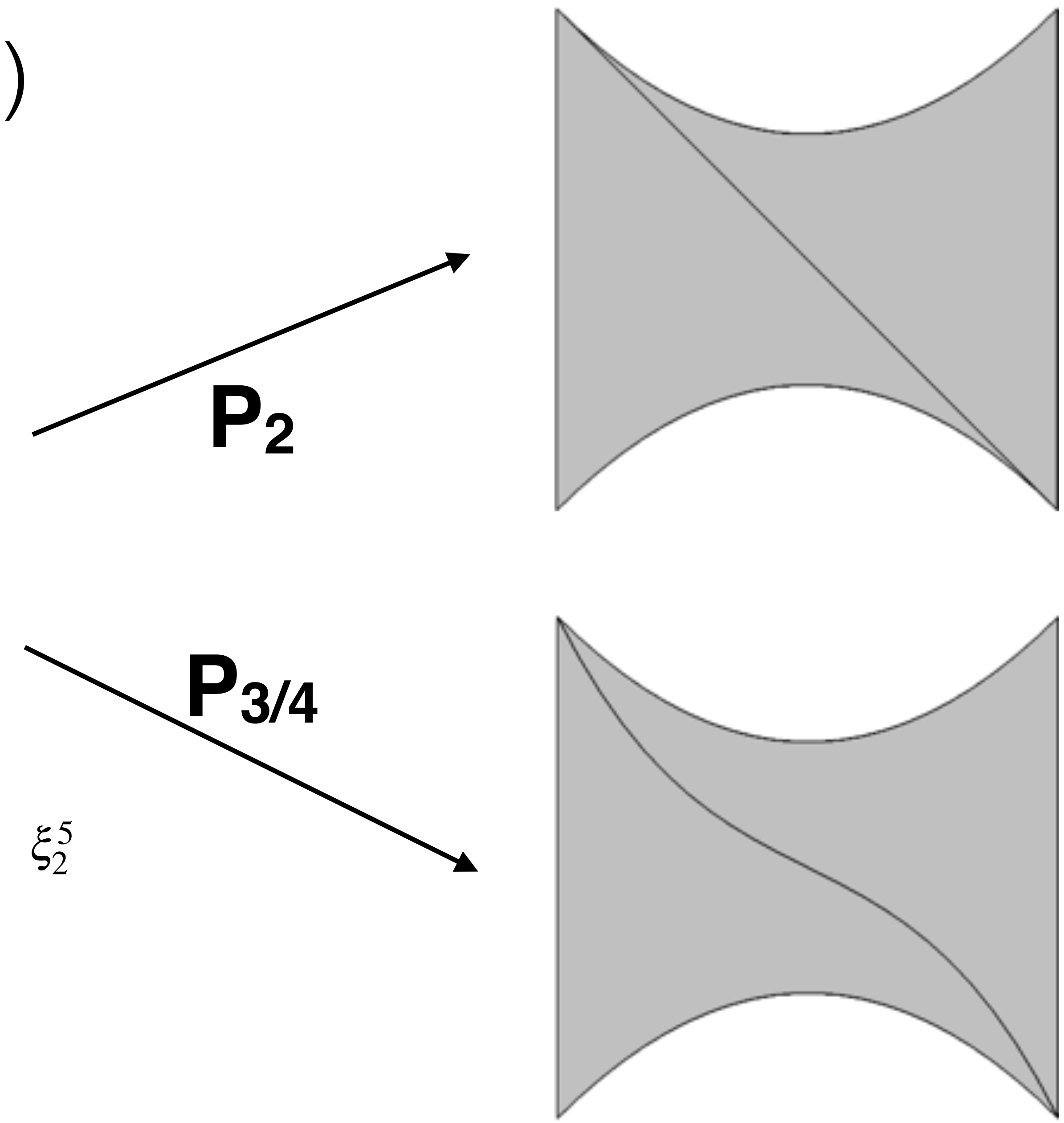
- Then pick polynomial space of target subelement so that it captures all polynomials of original mapping.

- Usually need to enrich subelements to support original mapping but depends on transform.

Why does this work? (2)

Spaces of quad (shaded) and triangle (outline)

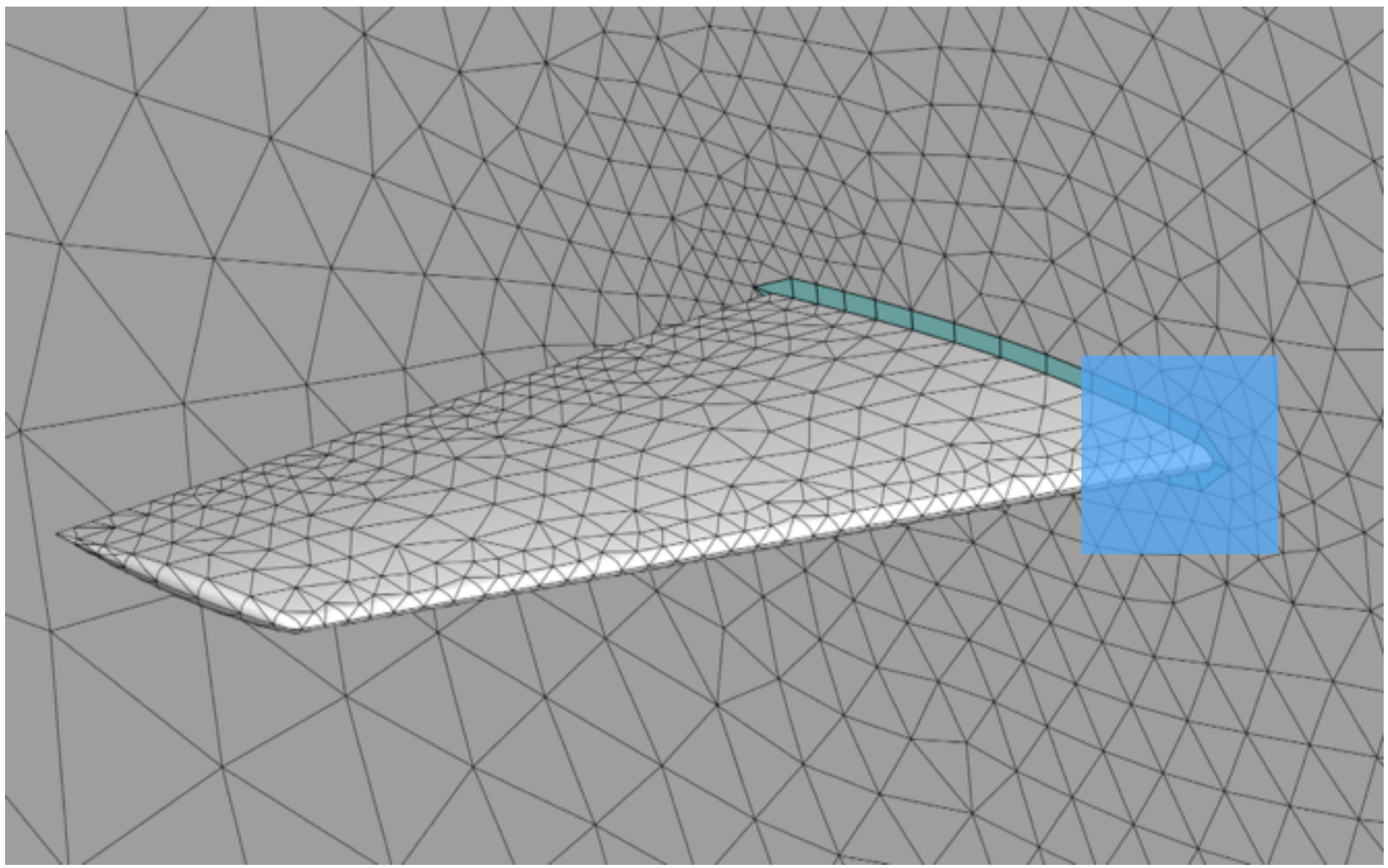




In general: order P quad → order 2P triangle

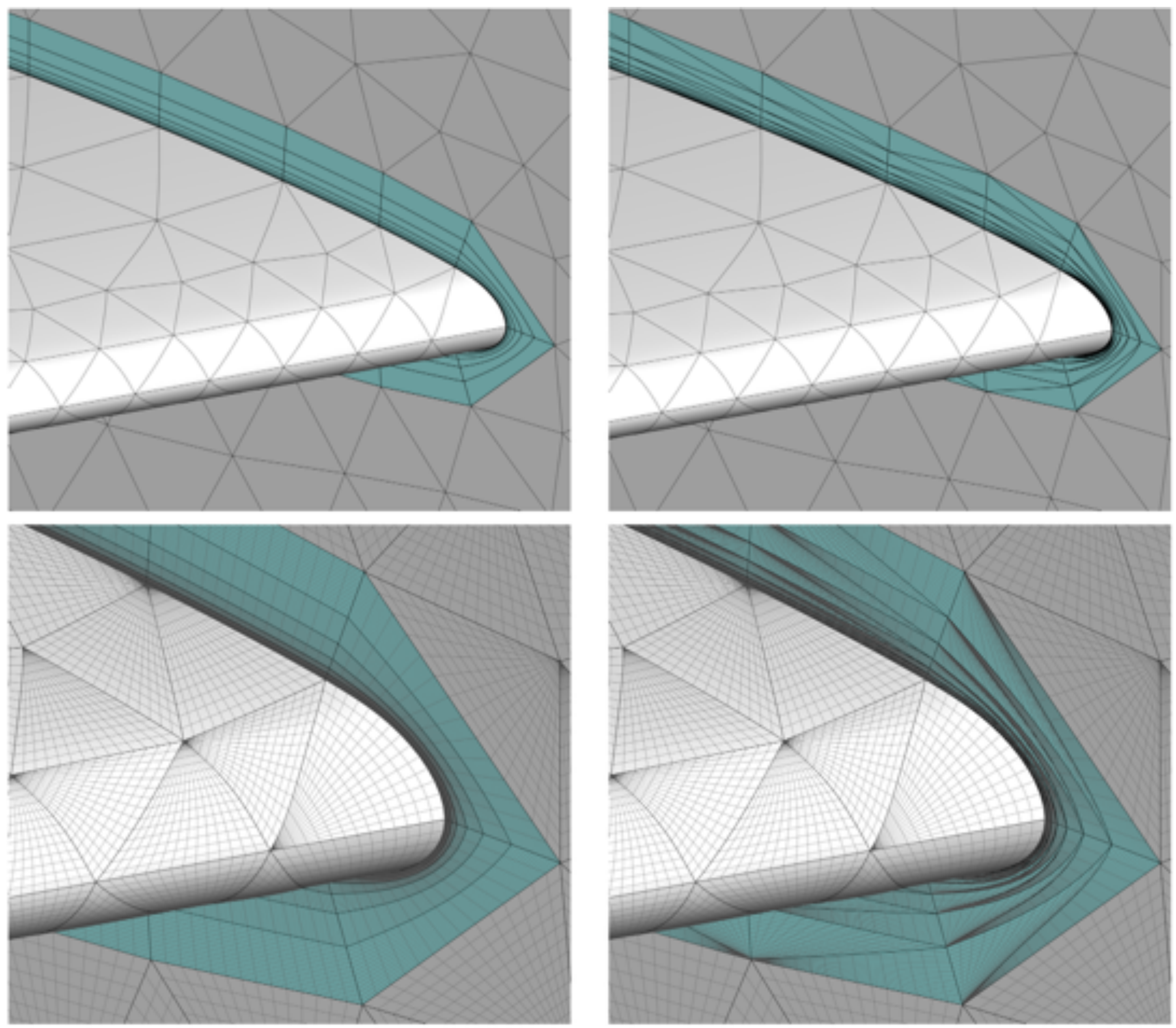
Curvature only in one direction → order P+1 triangle

ONERA M6 wing



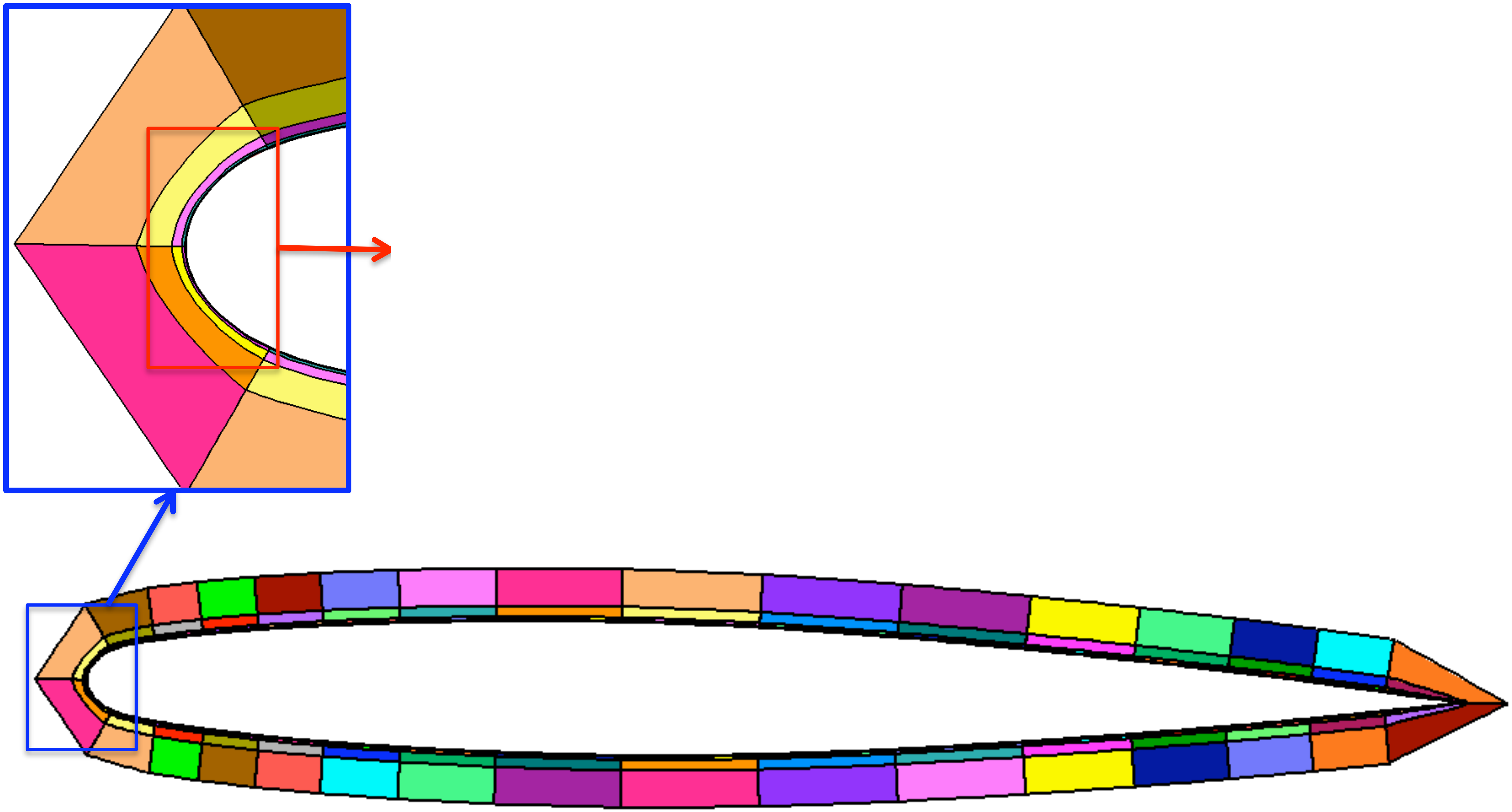
High polynomial order (P = 14)

ONERA M6 wing



Prisms Tets

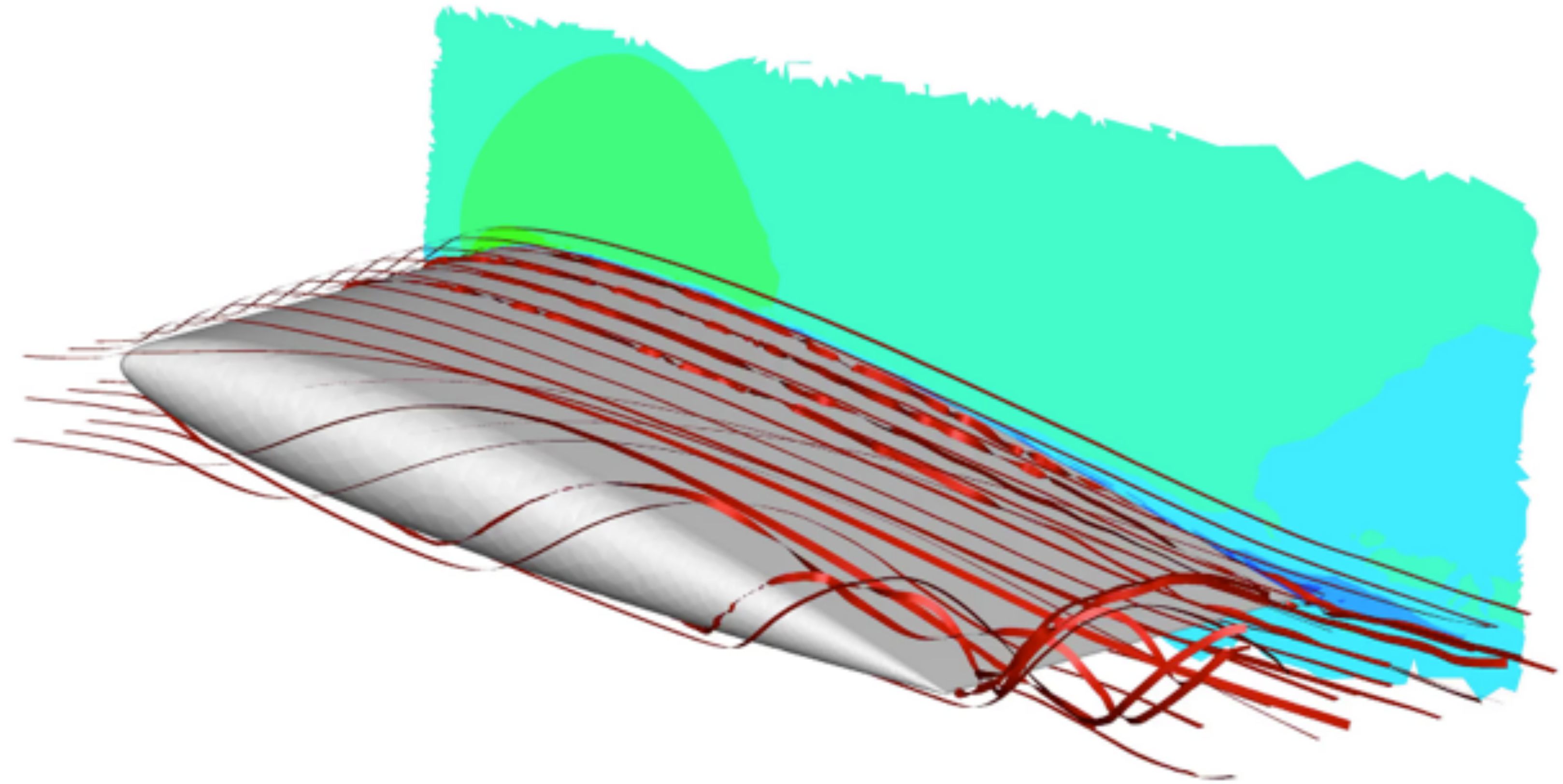
Proof of concept



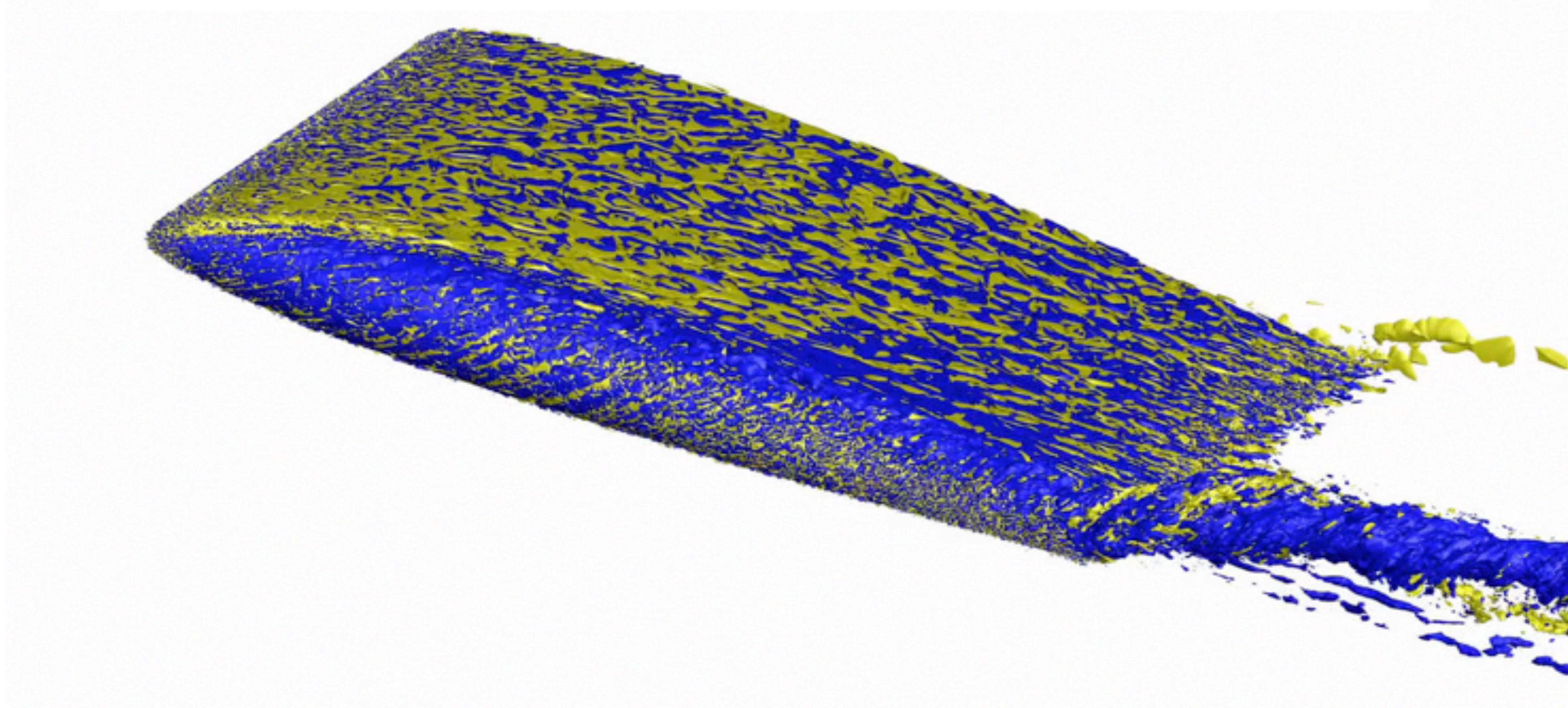
7 layers of refinement

NACA 0012 wing tip (Re = 1.2M)

Streamlines

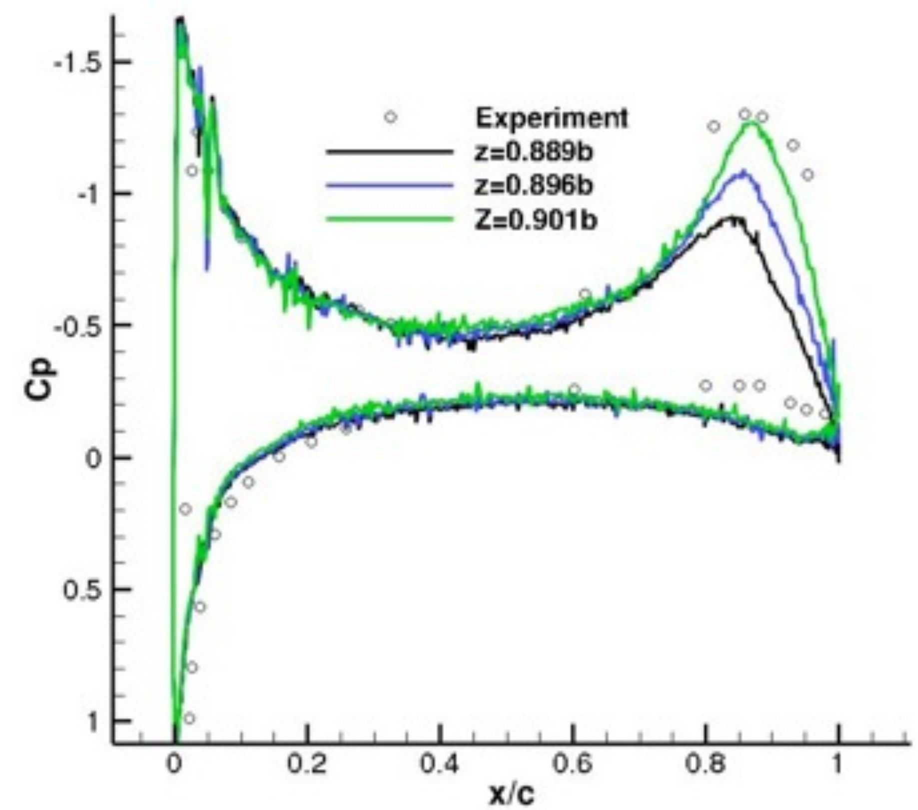


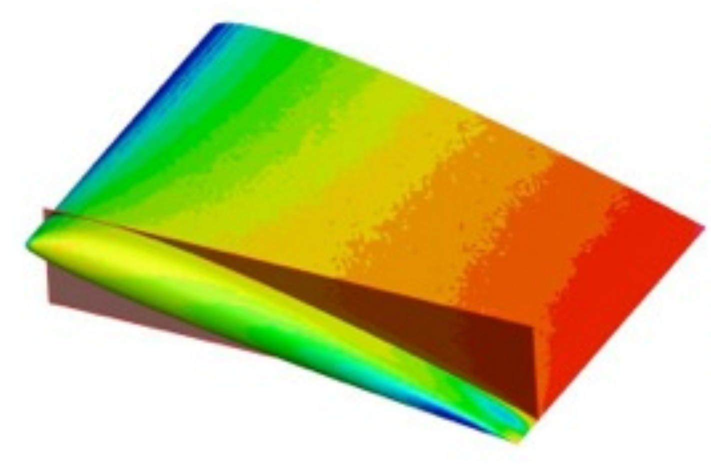
Streamwise vorticity



NACA 0012 comparisons

- DNS stabilised by using spectral vanishing viscosity (SVV) at polynomial order 5
- Wingtip vortex is captured
- Cut-plane of pressure distribution shows good agreement with experiment
- Still need to investigate effects of dealiasing on solution field





Conclusions

- High-order methods can be applied to these problems and successfully capture essential flow dynamics
- Still a need for high-order mesh generation strategies for coarse grid
- Future: grid deformation for coarse grid with emphasis on element quality

Thanks for listening!