High-order spectral/hp methods for aerodynamic applications

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Introduction

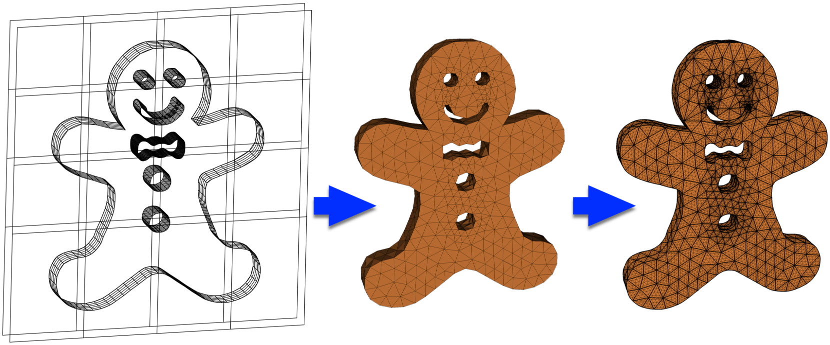
In this talk, we introduce two techniques designed to aid the implementation of high-order methods for industrial application:

- **High-order mesh generation:** generating curvilinear meshes for complex geometries, and a boundary layer refinement technique.
- ***h*-to-*p* efficiency:** a hybrid approach to operator evaluation to increase computational efficiency.

High-order mesh generation (*p*-mesh)

Steps:

1. CAD boundary representation (B-Rep).
2. Mesh of linear elements.
3. Mesh of high-order elements.



B-Rep Linear mesh High-order mesh

Producing meshes for high-Re simulations

Viscous flows $\rightarrow$ boundary layer around walls.

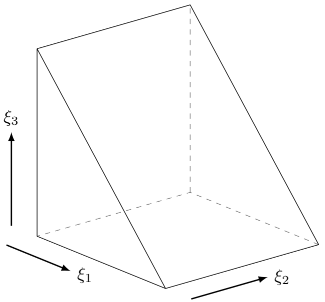
- From the surface triangulation, we generate a prismatic boundary layer (better mesh quality).
- Rest of the volume is constructed using unstructured tetrahedra.

For high Reynolds number simulations:

- Require an extremely thin boundary layer.
- Must not contain invalid elements.

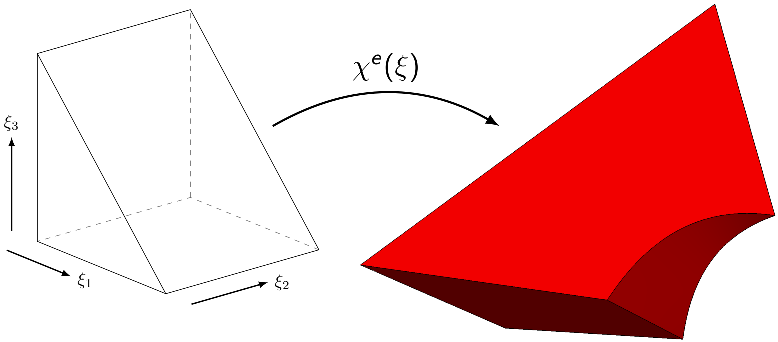
We introduce a method to refine a mesh with existing prismatic boundary layer to produce a valid prismatic or tetrahedralised mesh.

Boundary layer refinement



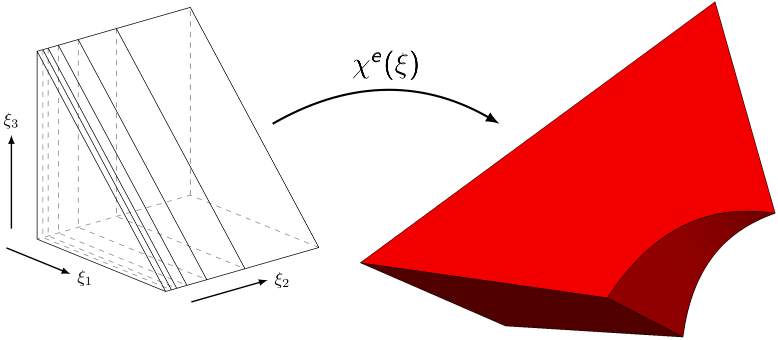
Begin with reference element Ω_{st}.

Boundary layer refinement



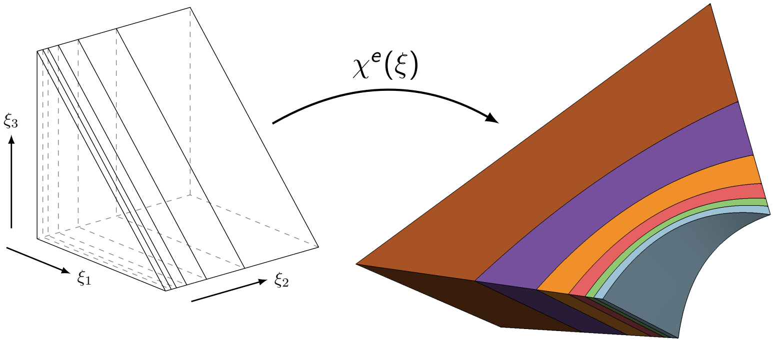
Apply mapping $\chi^e : \Omega_{st} \to \Omega^e$ to obtain Ω^e.

Boundary layer refinement



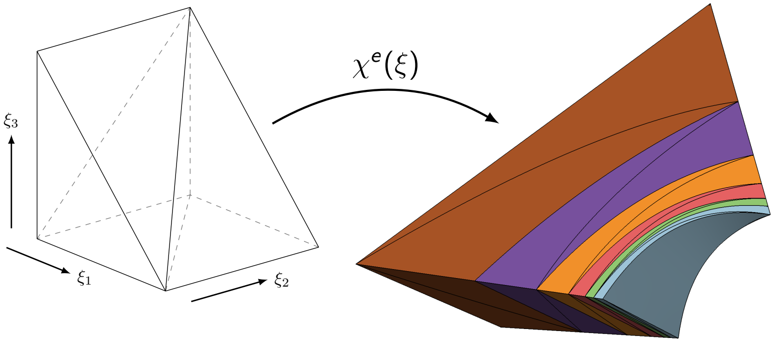
Split Ω_{st} into subelements according to a spacing function $f(\xi_2)$.

Boundary layer refinement



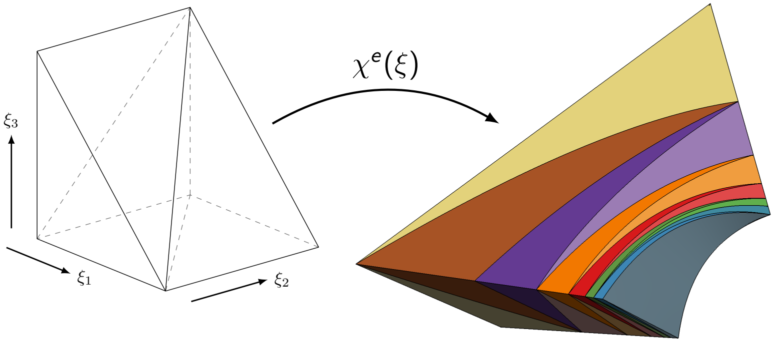
Apply χ^e to split Ω^e into curved subelements.

Boundary layer refinement



To get tetrahedral mesh, split each subelement of Ω_{st} into tetrahedra and apply χ^e.

Boundary layer refinement



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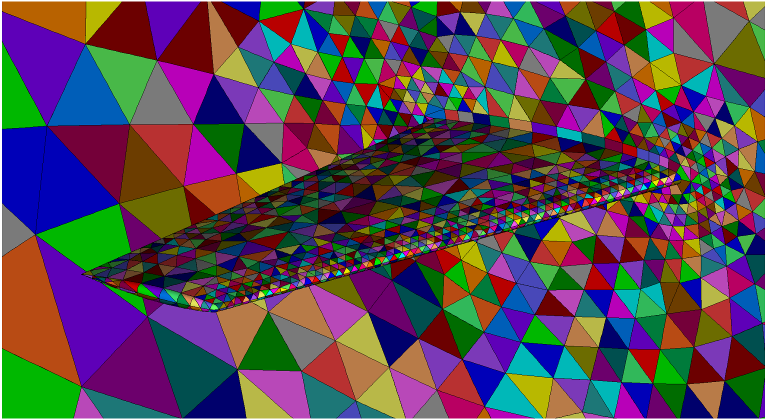
Example: ONERA M6 Wing

Boundary layer thickness: $y^+ \approx 10$ for $\mathrm{Re} = 10^5$.



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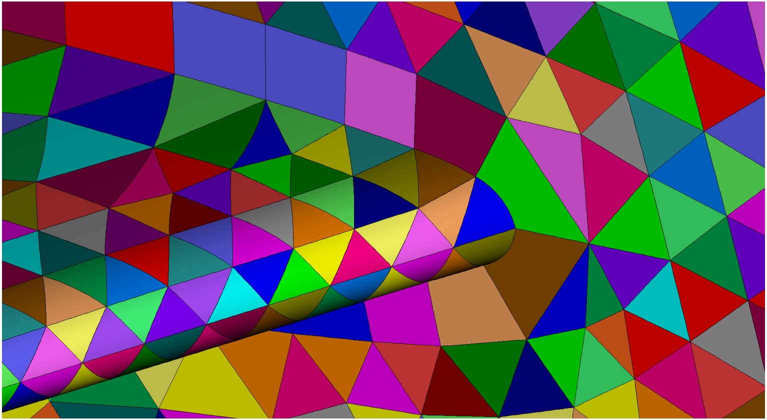

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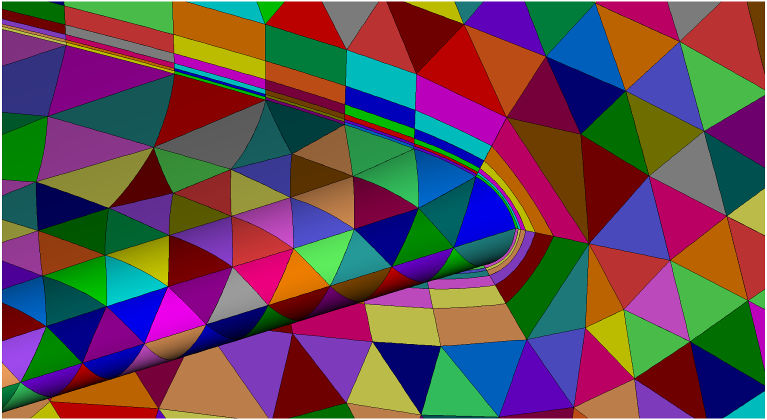

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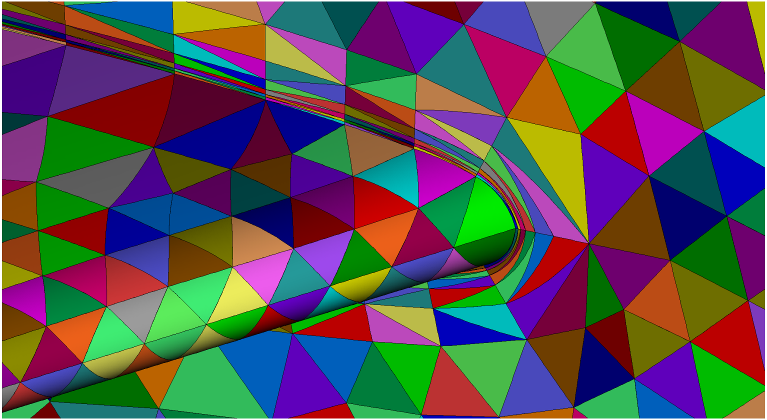

Boundary layer refinement

Features:

- Easy to implement: mapping χ^e is used for spectral element method.
- Produces valid meshes so long as $\det(\chi^e) > 0$.
- Any spacing function $f(\xi_2)$ can be used to determine sizes of subelements.

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- Easy to implement: mapping χ^e is used for spectral element method.
- Produces valid meshes so long as $\det(\chi^e) > 0$.
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Limitation:

- Assumes an existing valid coarse prismatic mesh.

Operator evaluation strategies

Consider the matrix resulting from the discretisation of an operator on a mesh of elements.

- Assume C^0 continuity of a function approximated on the mesh.
- This can imposed through establishing equality of coefficients representing connected vertices/edges/faces.

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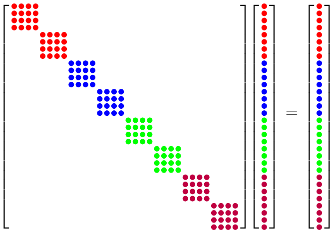
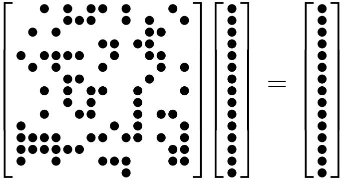
There are two well-known operator strategies which can be employed:



Local assembly



Global assembly

Operator evaluation strategies

Which is better? Depends on:

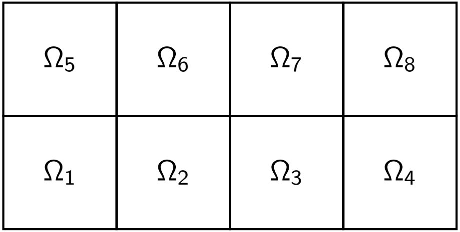
- Polynomial order;
- Architecture (CPU/GPU);
- Dimensionality of mesh.

Generally:

- low order ($P = 1, 2$) $\rightarrow$ global assembly
- high-order $\rightarrow$ local assembly (or sum-factorisation...)

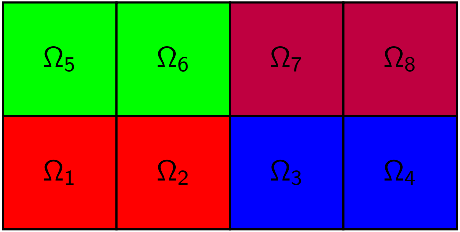
Question: Is there a more optimal strategy which combines a mixture of the two approaches?

Hybrid assembly



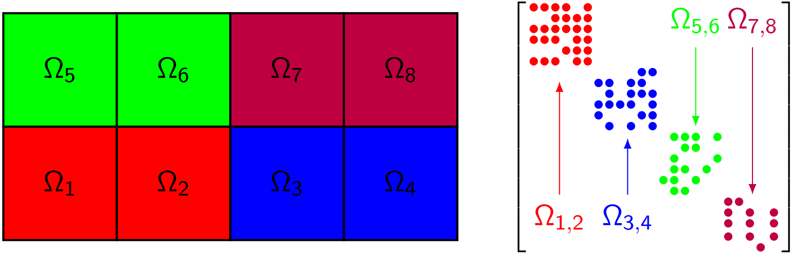
Start with a mesh, for example eight quadrilaterals.

Hybrid assembly



Choose a partition of the mesh.

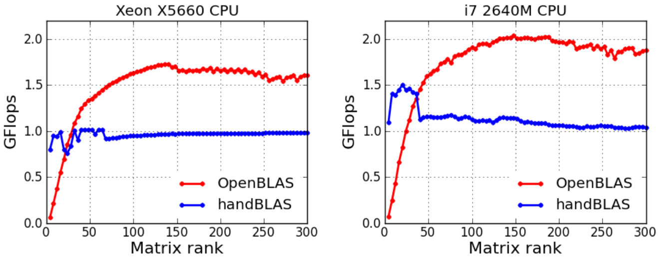
Hybrid assembly



Perform a ‘global’ assembly for each partition to create a *patch*, and constrct a ‘hybrid‘ block matrix.

Why do a hybrid approach?

With hybrid assembly, we can target and optimise for platform specific quantities; e.g. processor cache size.

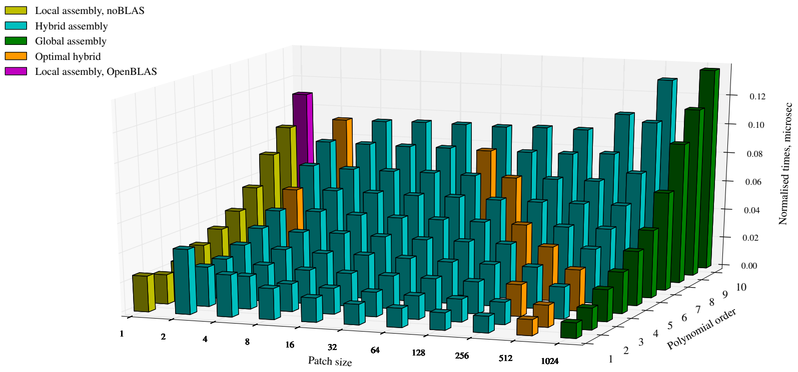


For a given polynomial order we target patch and matrix sizes which optimise CPU throughput using either OpenBLAS or a hand-coded routine.

Some benchmarks

Timings for matrix-vector multiplcation representing an operator on a 1024 triangle mesh were performed.

The following figure shows the optimal techniques for a variety of patch sizes and polynomial orders.



h-to-p efficiency

Generally these timings suggest that:

- Low-order $\rightarrow$ global assembly (1 patch);
- High-order $\rightarrow$ local assembly (N_{el} patches);
- For middle-sized polynomial orders there is some more optimal choice of patch size.

h-to-*p* efficiency

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- Low-order $\to$ global assembly (1 patch);
- High-order $\to$ local assembly (N_{el} patches);
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Choosing these parameters manually is difficult. *libFEMpp*:

- will be a platform- and code-independent library;
- combine system configuration and profiling data to produce optimal patch sizes for a given input mesh;
- is currently under development.

Conclusions

Mesh generation:

- Generation of arbitrary high-order meshes is a viable goal for complex geometries.
- Use of refinement technique aides in the generation of boundary layer meshes for aeronautically realistic Reynolds numbers.

h-**to**-*p* **efficiency:**

- Careful choice of operator evaluation strategies is essential for high-order elements.
- Hybrid assembly demonstrates an approach whereby this process can be automated and tuned for individual architectures.