High-order spectral/hp methods for aerodynamic applications

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ABSTRACT

The use of high-order methods, together with a combination of h- and p-refinement, offers the potential of achieving higher accuracy for a given computational cost when dealing with complex aeronautical geometries and their associated flows. However, before these methods are adopted by industry, there are a number of issues that must be addressed to meet industrial requirements. In this article we will describe our attempts towards addressing two of the main bottlenecks preventing the widespread industrial use of such methods: the generation of high-order curvilinear meshes for complex geometries and computational efficiency.

Our high-order mesh generator uses a CAD definition of the region to be meshed to generate a mesh of linear prisms in the boundary layer and tetrahedral elsewhere. The generation of a curvilinear mesh of high-order spectral/hp elements proceeds in a bottom-up fashion by generating additional nodes along the edges of the mesh, then on faces and finally in the elements. The location of the additional points is compatible with the CAD geometry and is obtained by minimising a measure of energy. To increase the robustness of the procedure we have incorporated a curvature-based criterion that reduces the appearance of invalid elements. We present meshes for some example geometries such as the ONERA M6 wing, which can be seen in figure 1, and also demonstrate how such meshes can be adapted for high Reynolds number simulations through the use of a boundary layer refinement technique.

In many industrial problems, the polynomial order of a numerical expansion is often determined by the desired computational error of the resulting approximation. For example, a sixth-order approxmation of a flow field on a coarse grid will yield a smooth and accurate realisation of the various structures contained within the flow. However, when this level of accuracy is not required, a finer grid coupled with a linear approximation which captures the main features of the flow may be more appropriate in terms of computational cost. The spectral element and hp finite element methods methods can be considered as bridging the gap between the – traditionally low-order – finite element method on one side and high-order spectral methods on the other side. Consequently, a major challenge which arises in implementing the spectral/hp element method is to design algorithms that perform efficiently for both low- and high-order discretisations, as well as discretisations in the intermediate regime.

In this presentation, we briefly explain how the judicious use of different implementation strategies can be employed to achieve high efficiency across a range of polynomial orders [1, 2]. Further, based upon this efficient implementation, we analyse which spectral/hp discretisation minimises the computational cost to solve elliptic steady and unsteady model problems up to a predefined level of accuracy.

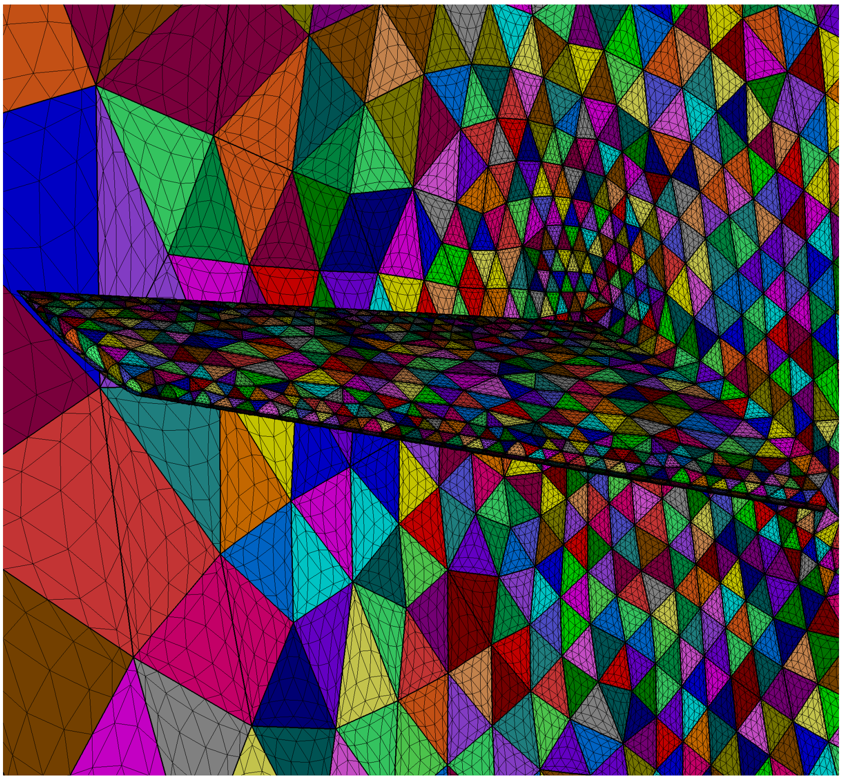

Figure 1: High-order surface triangulation of the ONERA M6 wing at polynomial order $P = 7$, showing both elemental boundaries and interior nodal points.

References

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