The onset of turbulence in pipe flow

David Moxey[1], Dwight Barkley[1],
Kerstin Avila, Alberto de Lozar, Marc Avila, Björn Hof

Mathematics Institute
University of Warwick[1]

British Applied Mathematics Conference,
13th April 2011

Introduction

- Understanding instability in fluid flow and, specifically, the transition to turbulence are well-known outstanding problems in fluid dynamics (see Reynolds, 1883).
- Pipe flow is geometrically simple, but exhibits complex transitional behaviour.
- This is dependent upon the dimensionless *Reynolds number:*

$$\mathrm{Re} = \frac{\overline{U}D}{\nu}$$

Introduction

- Understanding instability in fluid flow and, specifically, the transition to turbulence are well-known outstanding problems in fluid dynamics (see Reynolds, 1883).
- Pipe flow is geometrically simple, but exhibits complex transitional behaviour.
- This is dependent upon the dimensionless *Reynolds number:*

$$\mathrm{Re} = \frac{\overline{U}D}{\nu}$$

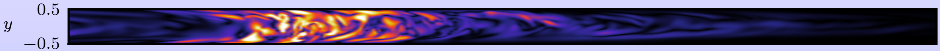
- However, the critical Reynolds number Re_c, above which turbulence becomes sustained, remains elusive.
- **Best guess:** $\mathrm{Re}_c \sim O(2{,}000)$

Large-scale structures

- For 1,700 $\leq \mathrm{Re} \leq$ 2,300, introducing a disturbance to a laminar fluid results in the formation of *puffs*.
- They are small pockets of turbulence which co-exist with laminar flow and are important in understanding transitional behaviour.

Large-scale structures

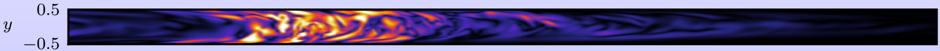
- For 1,700 $\leq \mathrm{Re} \leq$ 2,300, introducing a disturbance to a laminar fluid results in the formation of *puffs*.
- They are small pockets of turbulence which co-exist with laminar flow and are important in understanding transitional behaviour.



- This puff was recorded at $\mathrm{Re} =$ 2,250 and has length $L \approx 25D$, so larger pipe lengths are needed to capture their behaviour $\Rightarrow$ more computer effort needed.

Large-scale structures

- For 1,700 $\leq \mathrm{Re} \leq$ 2,300, introducing a disturbance to a laminar fluid results in the formation of *puffs*.
- They are small pockets of turbulence which co-exist with laminar flow and are important in understanding transitional behaviour.



- This puff was recorded at $\mathrm{Re} = 2{,}250$ and has length $L \approx 25D$, so larger pipe lengths are needed to capture their behaviour $\Rightarrow$ more computer effort needed.
- **Questions:**
  - How do puffs behave?
  - What role do they play in the transition process?

Puff decay

- Recent attempts to find Re_c have studied the decay of puffs for $\mathrm{Re} \leq 2{,}000$.
- Decay is a *stochastic* process. We consider the survival function

$$S(t) = \mathbb{P}(\text{puff does not decay for } 0 \leq t' \leq t)$$

Puff decay

- Recent attempts to find Re_c have studied the decay of puffs for $\mathrm{Re} \leq 2{,}000$.
- Decay is a *stochastic* process. We consider the survival function

$$S(t) = \mathbb{P}(\text{puff does not decay for } 0 \leq t' \leq t)$$

- To identify the distribution of S:
  - a simulation starts with an isolated puff;
  - observe the decay time for this puff (if any).

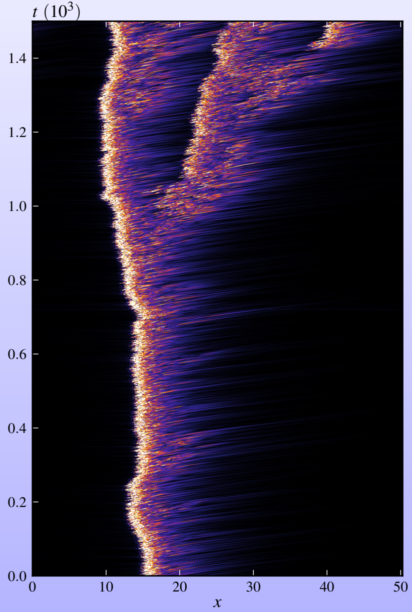
Puff decay

- Recent attempts to find Re_c have studied the decay of puffs for $\mathrm{Re} \leq 2{,}000$.
- Decay is a *stochastic* process. We consider the survival function

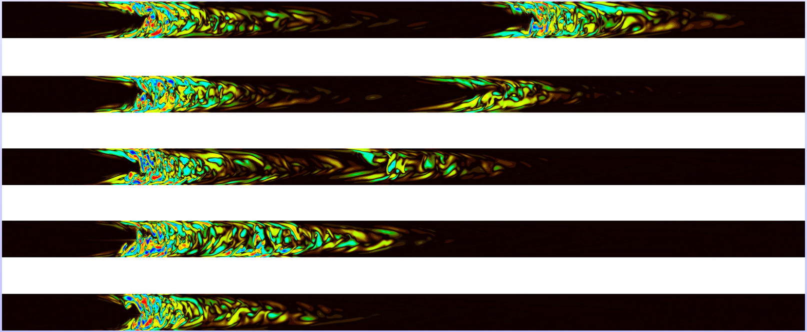
$$S(t) = \mathbb{P}(\text{puff does not decay for } 0 \leq t' \leq t)$$

- To identify the distribution of S:
  - a simulation starts with an isolated puff;
  - observe the decay time for this puff (if any).
- Decay occurs according to an exponential distribution:
  - decay is memoryless;
  - puffs have mean lifetime τ.
- Unfortunately, τ remains finite for any finite Re [Avila et al., 2010].

Puff splitting



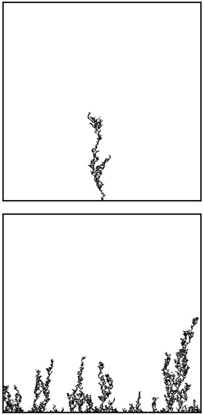
Puff splitting



These snapshots of streamwise vorticity $\nabla \times \boldsymbol{u}$ show the splitting process in action (courtesy of Marc Avila).

Spatio-temporal intermittency

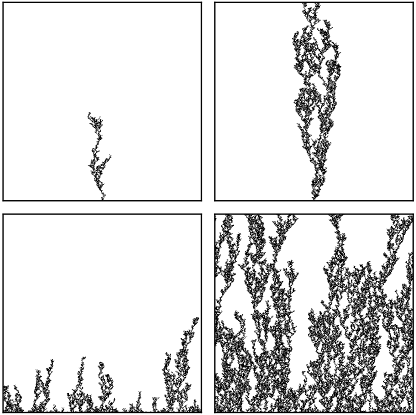
Many simple stochastic systems exhibit similar intermittent properties; for example, *directed bond percolation*.



$p < p_c$

Spatio-temporal intermittency

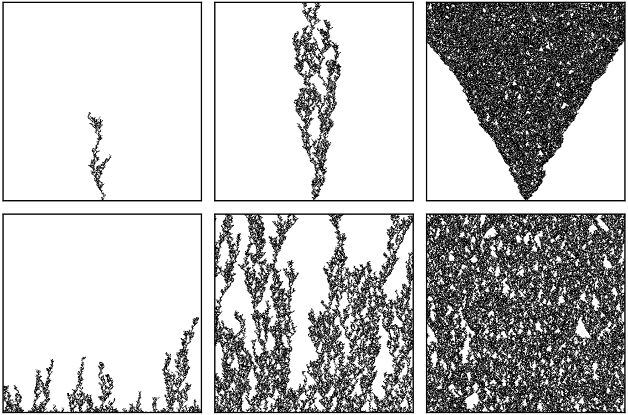
Many simple stochastic systems exhibit similar intermittent properties; for example, *directed bond percolation*.



$p < p_c$ $p = p_c$

Spatio-temporal intermittency

Many simple stochastic systems exhibit similar intermittent properties; for example, *directed bond percolation*.



$p < p_c$ $p = p_c$ $p > p_c$

Splitting statistics

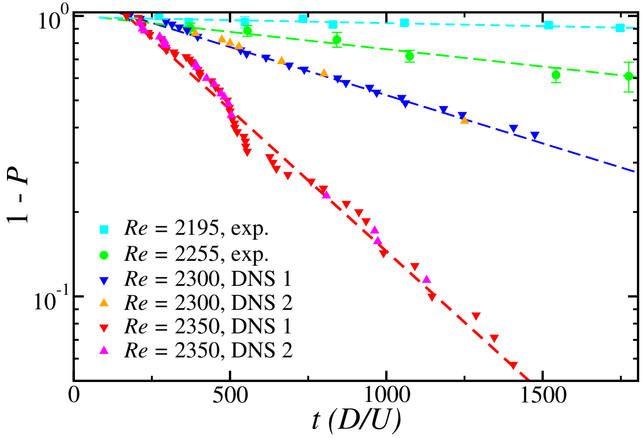
Observing puff splitting *statistically*, we calculate survival as

$$S(t) = 1 - P(t) = \mathbb{P}(\text{puff does not split for } 0 \leq t' \leq t)$$

Splitting statistics

Observing puff splitting *statistically*, we calculate survival as

$$S(t) = 1 - P(t) = \mathbb{P}(\text{puff does not split for } 0 \leq t' \leq t)$$



Lifetime statistics

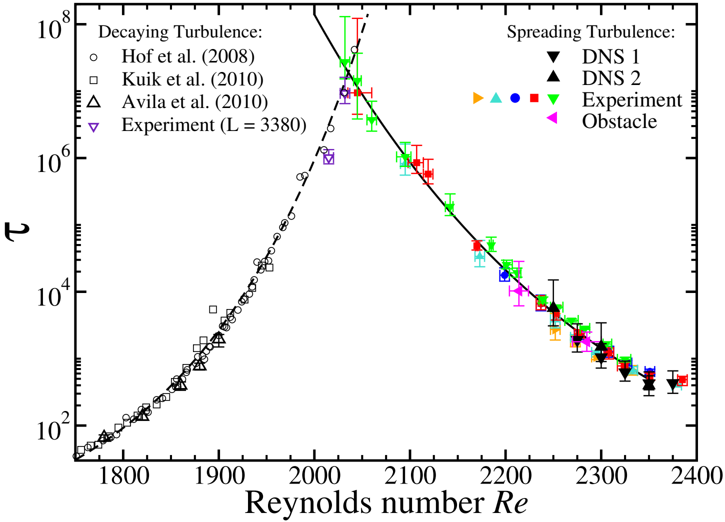
From each of these distributions, we see that

$$S(t) = \exp(-(t - t_0)/\tau)$$

- Like decay, splitting is a *memoryless* process.
- t_0 is the initial formation time – difficult to determine explicitly, but from observations we see that $t_0 \leq 170$.
- τ is the mean splitting time, and is a function of Re – estimated using a maximum likelihood estimator (MLE).

By varying Re, we calculate the density function and thus obtain τ as a function of Re.

Distribution of lifetimes



Experimental points (coloured) from Kerstin Avila's PhD work.

Conclusions

- $\mathrm{Re}_c = 2{,}040 \pm 10.$

Conclusions

- $\mathrm{Re}_c = 2{,}040 \pm 10$.
- The critical point has been difficult to determine because of the complex spatial coupling and the very long timescales involved.
- It denotes a phase transition from states which are more probable to decay to those which are more likely to split.

Conclusions

- $\mathrm{Re}_c = 2{,}040 \pm 10$.
- The critical point has been difficult to determine because of the complex spatial coupling and the very long timescales involved.
- It denotes a phase transition from states which are more probable to decay to those which are more likely to split.

Further work:

- Determining the underlying mechanism for splitting.
- Applying these techniques to other shear flows.
- Studies to determine an exact value for t_0.