The onset of turbulence in pipe flow

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Introduction

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- However, nobody thus far has been able to identify Re_c; the value beyond which turbulence becomes sustained.
- **Best guess:** $\mathrm{Re}_c \sim O(2{,}000)$

Large-scale structures

- For $1{,}900 \leq \mathrm{Re} \leq 2{,}300$, introducing a disturbance to a laminar fluid results in the formation of *puffs*.
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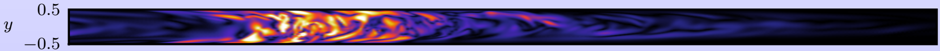

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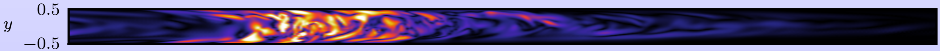

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- **Questions:**
  - How do puffs behave?
  - What role do they play in the transition process?

Standard methods

- To simulate the fluid, we perform a direct numerical simulation (DNS) of the underlying Navier-Stokes equations

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}\,.$$

- For increased accuracy, spectral-type schemes are preferred to finite-difference approaches.
- In these schemes, we approximate the solution fields by

$$u^\delta(\boldsymbol{x}, t) = \sum_{k=0}^{N-1} \hat{u}_k(t)\phi_k(\boldsymbol{x})$$

  where ϕ_k are referred to as *modes*.
- A typical spectral scheme for pipe flow uses:
  - Cylindrical polar co-ordinate system;
  - Fourier collocation points in azimuthal and axial directions;
  - Chebyshev in radial direction.

Numerical Methods

- Our simulations use an existing open-source code `Semtex`, which decomposes the domain Ω into quadrilateral *elements* Ω^e so

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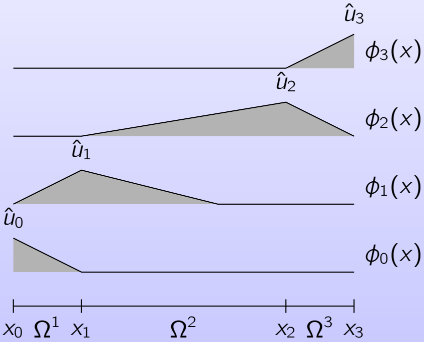
- This scheme allows for ‘nicer’ placement of nodal points: the typical scheme leads to a build-up of points around the axis.
- DNS is performed using a high-order splitting scheme of $O(\Delta t^2)$ or $O(\Delta t^3)$ using high-order pressure boundary condition at walls.

Spectral-element modes



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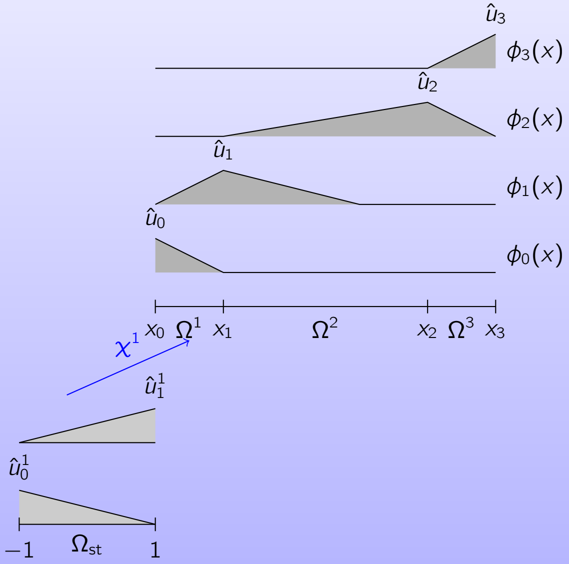

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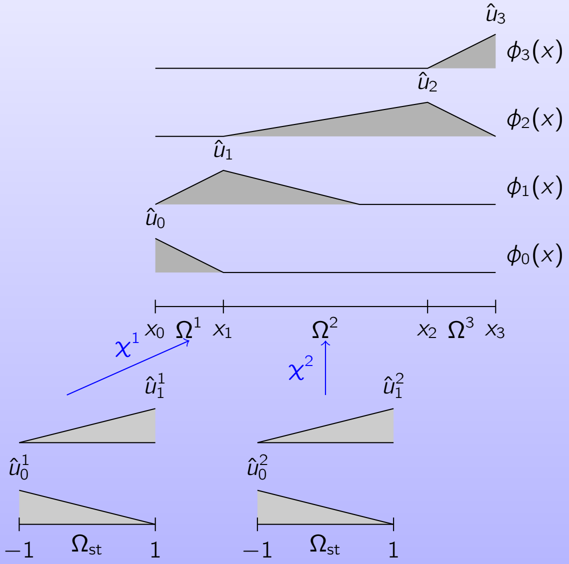

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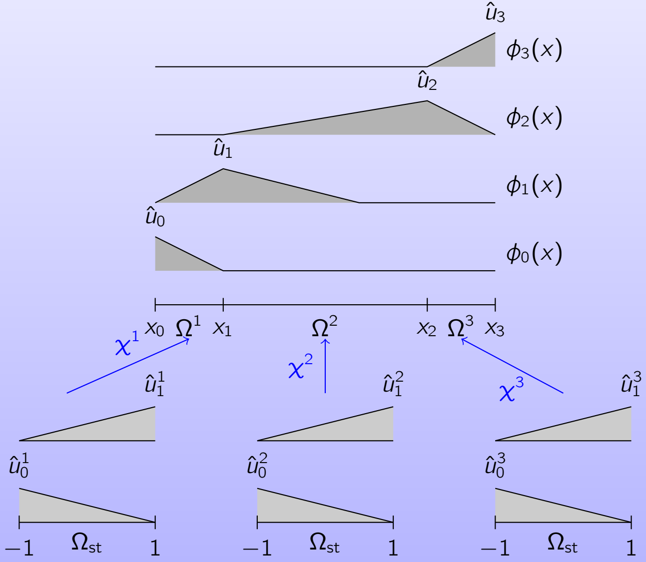

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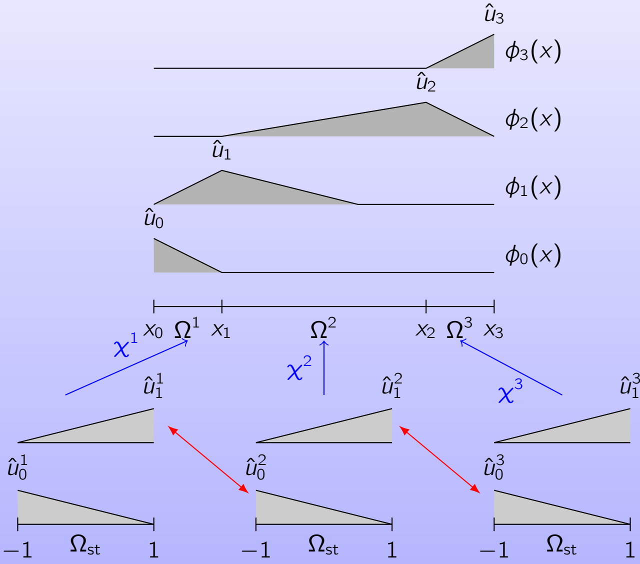

Resolution requirements

- Here we use Cartesian co-ordinates, so that 2D circular cross-sections use spectral element meshes.

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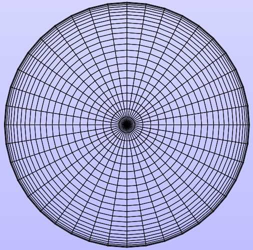

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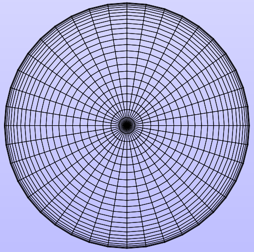

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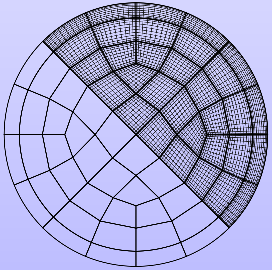

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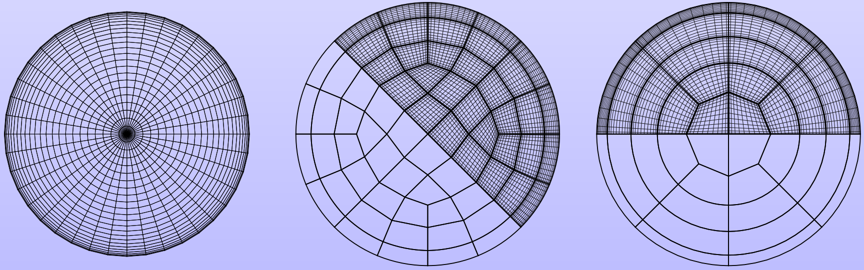
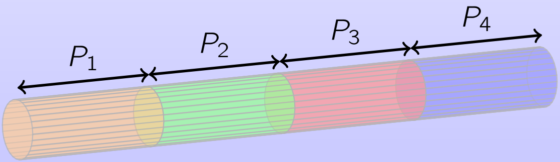


$N_r = 40, N_\theta = 46$ $\quad$ $N_{\mathrm{el}} = 64, P = 12$ $\quad$ $N_{\mathrm{el}} = 38, P = 12$

Computing requirements

- For $L = 25D$ pipe, need 384-512 axial nodes for $\mathrm{Re} \leq 3000$.
- In the scheme here, we parallelise over the axial direction.
- Pseudo-spectral scheme in axial direction $\Rightarrow$ parallel FFT.



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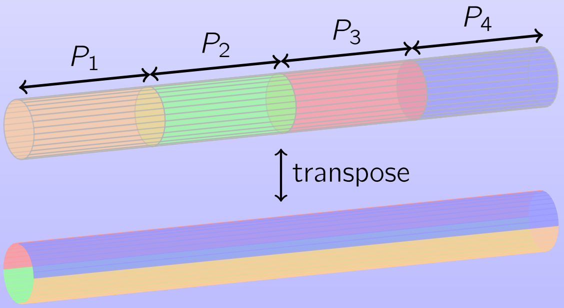

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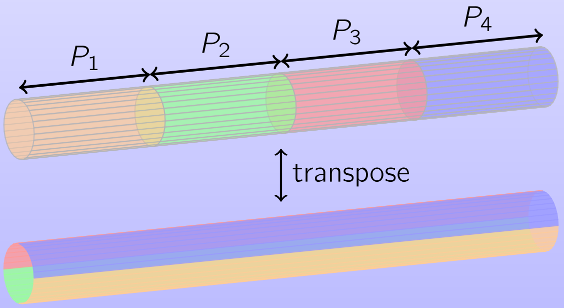


- Transpose requires all-to-all communication $\Rightarrow$ sad face.

Simulations

- The simulation starts with uniform turbulence using a short run at $\mathrm{Re} = 5{,}000$. We then reduce Re as follows:

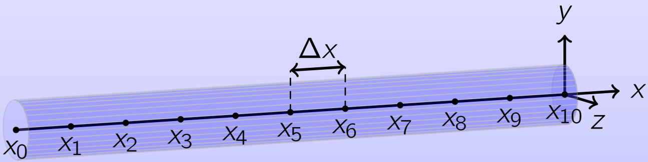
$$\begin{array}{lr} 5{,}000 \to 4{,}000 \to 3{,}000 & \text{500 time units} \\ 2{,}800 \to 2{,}600 & \text{600 time units} \\ 2{,}500 \to 2{,}450 \to 2{,}400 \to 2{,}350 \to \cdots & \text{2,000 time units} \end{array}$$

- Parameters for the simulations were:

L	$40\pi D \approx 125D$
N_x	2,048
N_{proc}	256
N_{el}	64
P	10
Re	$2{,}250 \leq \mathrm{Re} \leq 2{,}800$
$\overline{U}$	1
Δt	2×10^{-3}

History Plots

- Field data is too large to record regularly. We measure *history data*: every 0.1 time units, sample $\boldsymbol{v}$ and p from points along the pipe axis.



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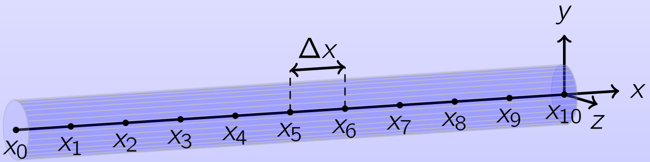
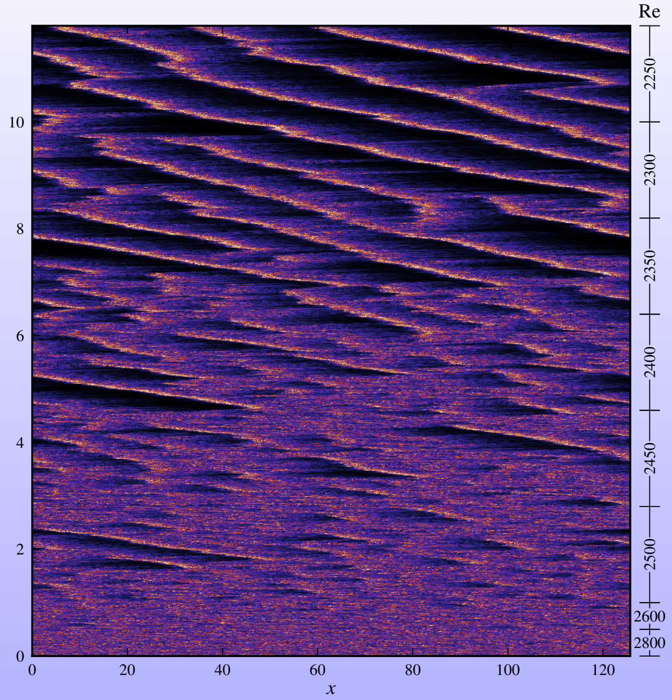


- For the velocity field $\boldsymbol{u} = (u, v, w)$, we construct the quantity

$$q(x,t) = \sqrt{v^2 + w^2}\Big|_{(x,y=0,z=0,t)} = \sqrt{2E_{\text{transverse}}}$$

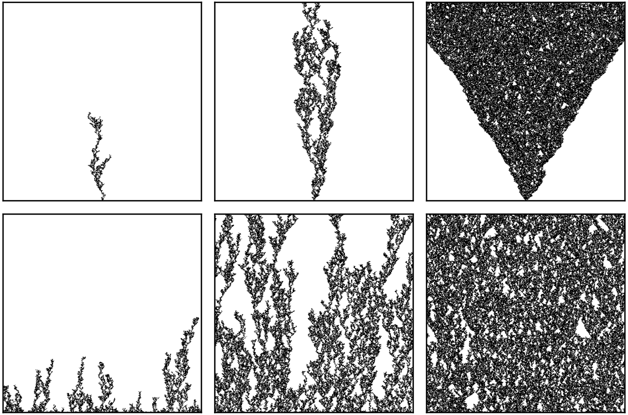
and then change to a moving frame of reference by the transformation

$$q(x,t) \to q(x - \overline{U}t, t).$$


Re
2250
2300
2350
2400
2450
2500
2600
2800
10
8
6
4
2
0
0
20
40
60
80
100
120
x

Spatio-temporal intermittency

Many simple stochastic systems exhibit intermittency over time. A particularly famous example is that of *directed bond percolation*:



$p < p_c$ $\quad$ $p = p_c$ $\quad$ $p > p_c$

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- Unfortunately, τ remains finite for any finite Re – this suggests that turbulence is perhaps always transient.

Puff splitting

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- The space-time figures show many examples of complex spatio-temporal intermittency.
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  - are there any laws regarding splitting which we can derive easily?
  - what is the underlying mechanism?
- It turns out that puff splitting is *also* a stochastic process. Therefore we must calculate distributions:

$$S(t) = 1 - P(t) = \mathbb{P}(\text{puff does not split for } 0 \leq t' \leq t)$$

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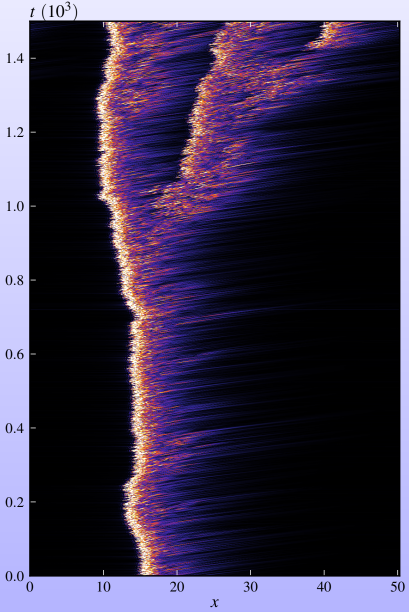

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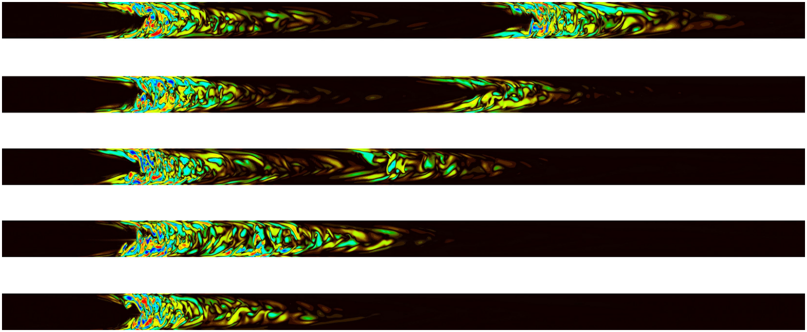
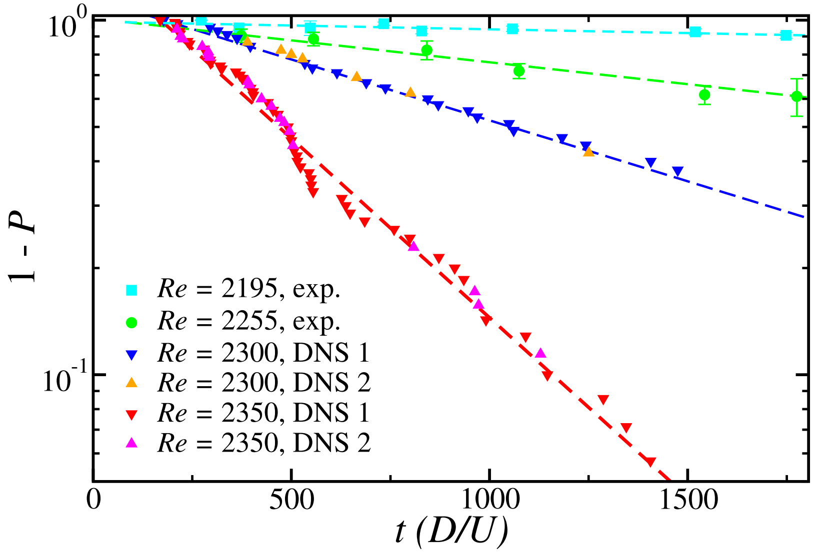


These snapshots of streamwise vorticity $\nabla \times \boldsymbol{u}$ show the splitting process in action.

Probability density functions



Lifetime statistics

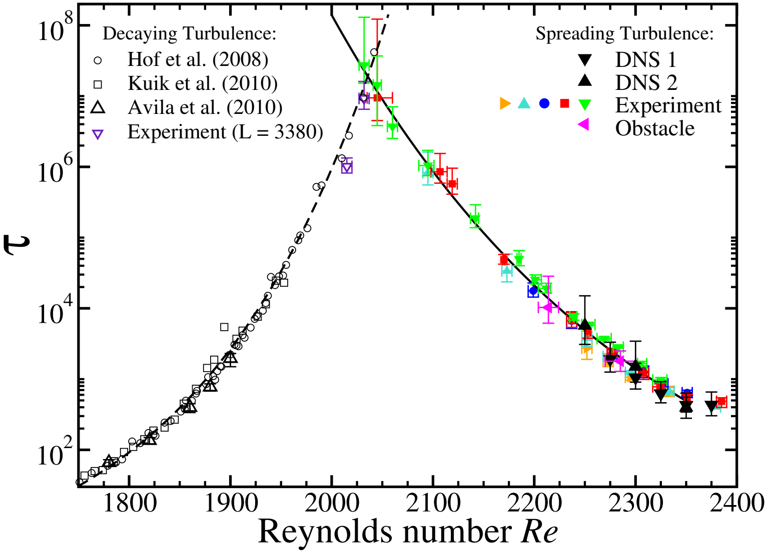
From each of these distributions, we see that

$$S(t) = \exp(-(t - t_0)/\tau)$$

- Like decay, splitting is a *memoryless* process.
- t_0 is the initial formation time.
- It is hard to determine explicity, but from observations we see that $t_0 \leq 170$.
- τ is the mean splitting time, and is a function of Re.

By varying Re, we calculate the density function and thus obtain the value of τ.

Distribution of lifetimes



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Further work:

- Further studies to determine an exact value for t_0.
- Determining the underlying mechanism for splitting.