Investigating pipe flow using a spectral element method

Studying large length scale structures in pipe flow

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Introduction

- Understanding instability in fluid flow and, specifically, the transition to turbulence are well-known outstanding problems in fluid dynamics (see Reynolds, 1883).
- The simple geometry of pipe flow provides a compelling environment to study transition.
- Theoretical approach is extremely complex; this problem is an ideal candidate for numerical simulation.
- Generally study of the transition problem has concentrated on investgations of **laminar to turbulent** flow.
- We wish to classify states found in the transition from **turbulent to laminar** flow.
- In particular, we are interested in discovering states involving **laminar-turbulent co-existance**.
- Thanks to CSC for computer usage – in particular the IBM cluster.

Large-scale structures

- For $1900 \leq \mathrm{Re} \leq 2300$, introducing a disturbance to a laminar fluid results in the formation of *puffs*.
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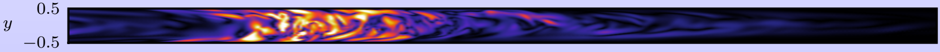

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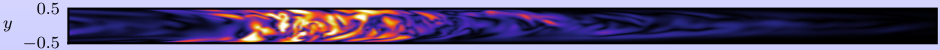

- This puff was recorded at $\mathrm{Re} = 2250$ and has length $L \approx 25D$, so larger pipe lengths are needed to capture their behaviour.
- **Questions:**
    - Do puffs occur naturally in the transition from turbulent to laminar flow?
    - What other states and transitions can we find?

Standard methods

- To study the transition problem we will perform a direct numerical simulation (DNS) of the underlying Navier-Stokes equations.
- For increased accuracy, spectral-type schemes are preferred to finite-difference approaches.
- Typical spectral scheme:
  - Cylindrical polar co-ordinate system;
  - Fourier collocation points in azimuthal and axial directions;
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- Efficient through use of FFT.
- Leads to a large build-up of nodal points at the origin.

Numerical Methods

- Our simulations use an existing open-source DNS code, `Semtex` [Blackburn & Sherwin, *J. Comput. Phys.*, 2004].
  - Designed as a 2D spectral element code.
  - Domain broken down into quadrilateral elements.
  - Each element has a set of local basis functions; generally polynomials of order P.
  - Third (homogeneous) dimension uses a Fourier pseudo-spectral method $\Rightarrow$ periodicity in third dimension.
  - This scheme allows for 'nicer' placement of nodal points.
- DNS is performed using a high-order splitting scheme of $O(\Delta t^2)$ or $O(\Delta t^3)$ using high-order pressure boundary condition at walls.
- Also supports both Cartesian and polar co-ordinate systems.

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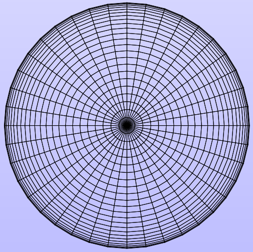

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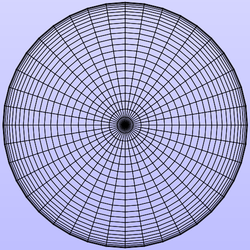

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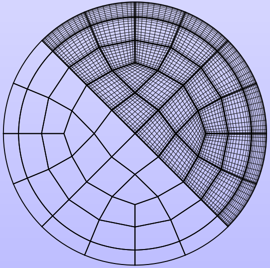

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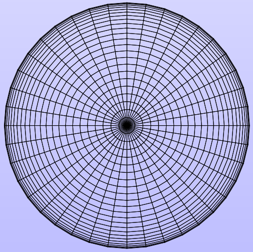

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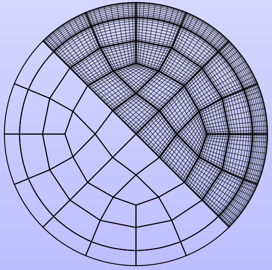

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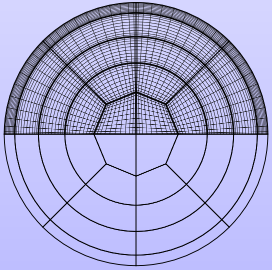
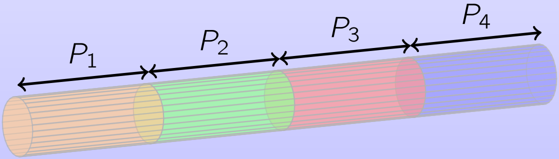


$N_{\mathrm{el}} = 38, P = 12$

Computing requirements

- For $L = 25D$ pipe, need 384-512 axial nodes for $\mathrm{Re} \leq 3000$.
- In the scheme here, we parallelise over the axial direction.
- Pseudo-spectral scheme in axial direction $\Rightarrow$ parallel FFT.



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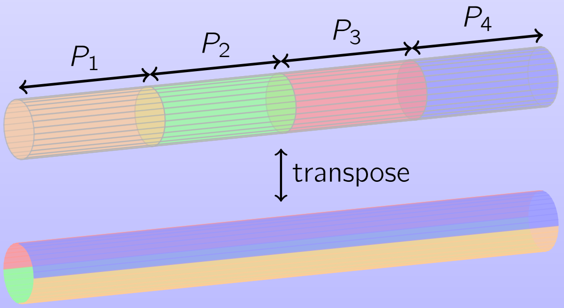

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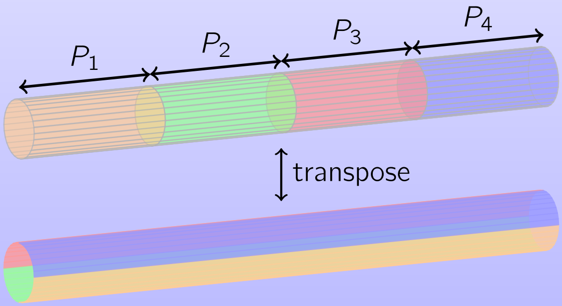


- Transpose requires all-to-all communication.

Simulations

- Parameters for the simulations were:

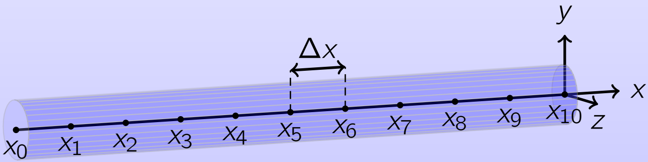
L	$40\pi D \approx 125D$
N_x	2048
N_{proc}	256
N_{el}	64
P	12
Re	$2250 \leq \mathrm{Re} \leq 2800$
Δt	2×10^{-3}

- The simulation is started with uniform turbulence using a short run at $\mathrm{Re} = 5000$ with increased resolution. We then reduce Re as follows:

$5000 \to 4000 \to 3000$ 500 time units
$2800 \to 2600$ 600 time units
$2500 \to 2450 \to 2400 \to 2350 \to \cdots$ 2000 time units

History Plots

- Field data is too large to record regularly. We measure *history data*: every 0.1 time units, sample $\boldsymbol{v}$ and p from points along the pipe axis.



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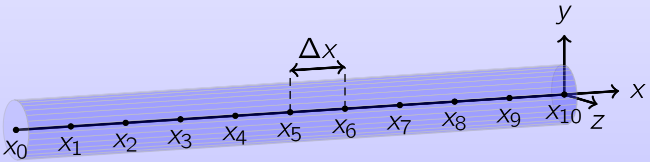
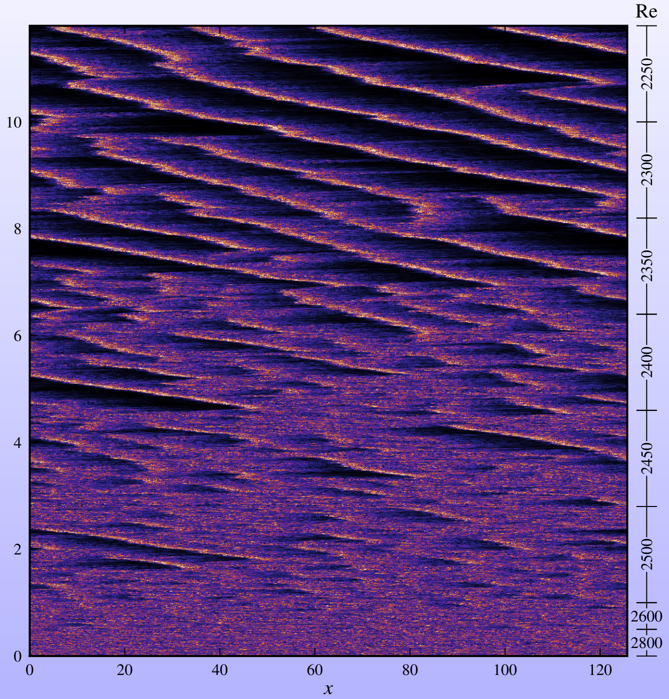


- For the velocity field $\boldsymbol{u} = (u, v, w)$, we construct the quantity

$$q(z,t) = \sqrt{v^2 + w^2}\Big|_{(x, y=0, z=0, t)} = \sqrt{2E_{\text{transverse}}}$$

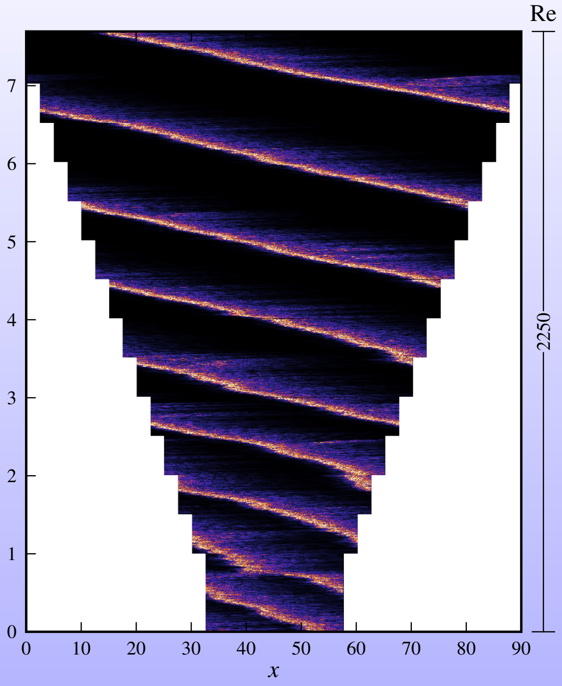
and then change to a moving frame of reference by the transformation

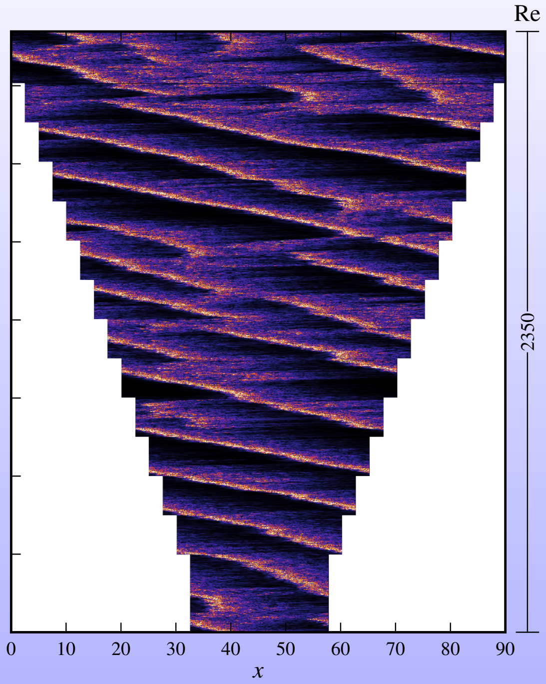
$$q(x,t) \to q(x - U_B t, t).$$


Re
2250
2300
2350
2400
2450
2500
2600
2800
10
8
6
4
2
0
0
20
40
60
80
100
120
x

Domain Expansion

- Space-time plots show that puffs split apart often in the $\mathrm{Re} = 2050$ case, but very little at $\mathrm{Re} = 2000$. Is this natural behaviour of Navier-Stokes?
- To answer the question, we consider a puff placed in a pipe of length $L = 25D$.
- Then we expand the domain every 500 time units by $L = 5D$ until we reach $L = 100D$.
- N_x is increased with L so that the domain remains correctly resolved.
- This was done for two separate simulations at both $\mathrm{Re} = 2000$ and $\mathrm{Re} = 2050$ using the same initial condition.
- Again we plot the history data with the quantity q.


Re
2250
0
1
2
3
4
5
6
7
0
10
20
30
40
50
60
70
80
90
x


Re
2350
0
10
20
30
40
50
60
70
80
90
x

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- **Future work:**
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  - Further investigate mechanisms for transition at Re_1 and Re_2.