Numerical studies of the transition to turbulence in long pipes

Studying large length scale structures

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Introduction

- Understanding instability in fluid flow and, specifically, the transition to turbulence are well-known outstanding problems in fluid dynamics.
- Pipe flow provides an ideal setting to study transition due to the simple geometry of the problem.
- Generally study of the transition problem has concentrated on investgations of **laminar to turbulent** flow. We wish to classify states found in the transition from **turbulent to laminar** flow.
- In particular, we are interested in discovering states involving **laminar-turbulent co-existance**.
- This talk outlines results that we have obtained from DNS of the pipe flow problem at transitional Reynolds numbers.

Large-scale structures

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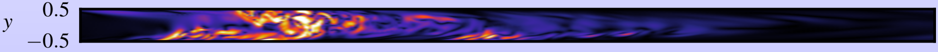

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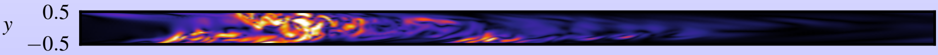


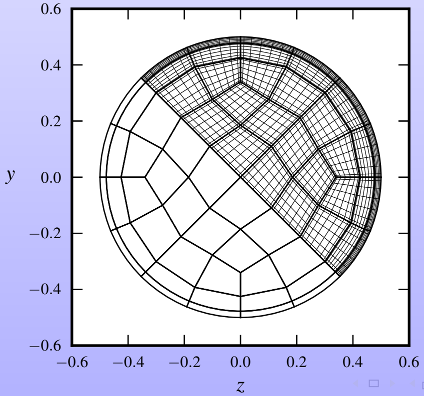
- This puff was recorded at $\mathrm{Re} = 2000$ and has length $L \approx 25D$, so larger pipe lengths are needed to capture their behaviour.
- **Questions:**
  - Do puffs occur naturally in the transition from turbulent to laminar flow?
  - What other states and transitions can we find?

Numerical Methods

- Our simulations use an existing open-source DNS code, `Semtex` [Blackburn & Sherwin, *J. Comput. Phys.*, 2004].
- `Semtex` is specifically a 2D spectral element code, but is extended to three dimensions by using a Fourier pseudo-spectral method ⇒ periodicity in third dimension.
- DNS is performed using a high-order splitting scheme incorporating high-order pressure boundary condition at walls.
- Can be used under both Cartesian and cylindrical co-ordinate systems.
- Parallelised using MPI (requires a parallel FFT).

Problem framework

- For the results presented here, we use a Cartesian co-ordinate system:
  - → Spectral elements in circular cross-sections.
  - → Periodic in axial direction.
- Allows for better placement of elements around the pipe wall.



Simulations

- Parameters for the simulations were:

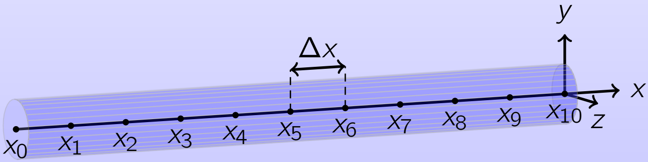
L	$16\pi D \approx 50D$	$48\pi D \approx 150D$
N_x	768	2048
N_{proc}	64	128
Re	$1900 \leq \mathrm{Re} \leq 2150$	$2000 \leq \mathrm{Re} \leq 2500$
Δt	2×10^{-3}	2×10^{-3}

- The flow is driven using a constant volumetric flux $q = 1 \Rightarrow U_B$ is fixed.
- The simulation is started with uniform turbulence using a short run at $\mathrm{Re} = 5000$ with increased resolution. We then reduce Re as follows:

$5000 \to 4000 \to 3000$	500 time units
$2800 \to 2650 \to 2500 \to 2350$	1000 time units
$2200 \to 2150 \to 2100 \to 2050 \to \cdots$	2000 time units

History Plots

- Field data is too large to record regularly. We measure *history data*: every 0.1 time units, sample $\boldsymbol{v}$ and p from points along the pipe axis.



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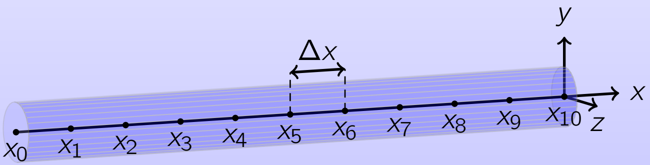
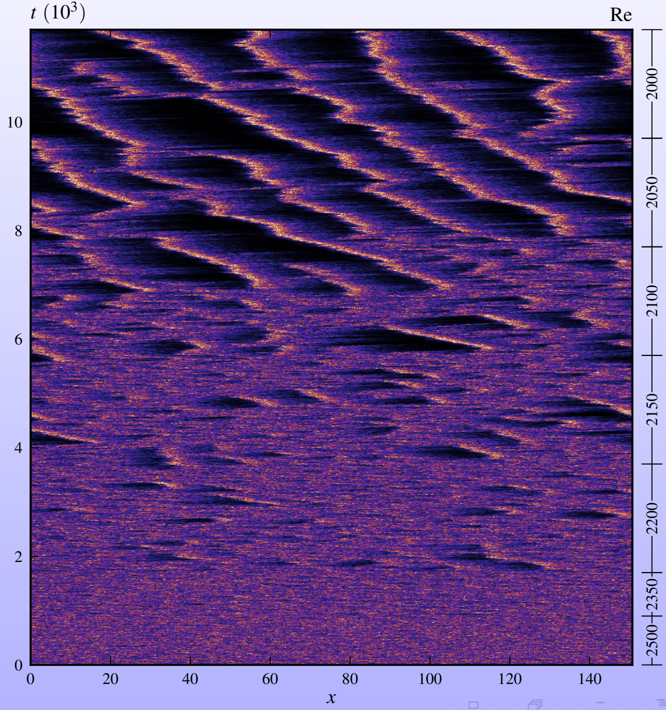


- For the velocity field $\boldsymbol{u} = (u, v, w)$, we construct the quantity

$$q(z,t) = \sqrt{v^2 + w^2}\Big|_{(x,y=0,z=0,t)} = \sqrt{2E_{\text{transverse}}}$$

and then change to a moving frame of reference by the transformation

$$q(x,t) \to q(x - U_B t, t).$$



t (10^3)
Re
10
8
6
4
2
0
0
20
40
60
80
100
120
140
x
2000
2050
2100
2150
2200
2350
2500

Summary of space-time data

The space-time plot gives a good overview of some of the interesting phenomenon seen with the reduction in Re. In particular, the events we will focus on are:

- Transition from uniform turbulence to intermittence at $\mathrm{Re} = 2200$ which is clearly observed.
- A not-so-clear transition involving puff splitting at $\mathrm{Re} = 2050$.

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Extended simulations were performed at transitional Re with $L = 25D$ to gather more accurate statistics.

- Four independent initial conditions were used in separate simulations.
- At each Reynolds number, each of the simulations were run for 4,000 time units;
- Total 16,000 time units for each Re.

Intermittency factor

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Definition

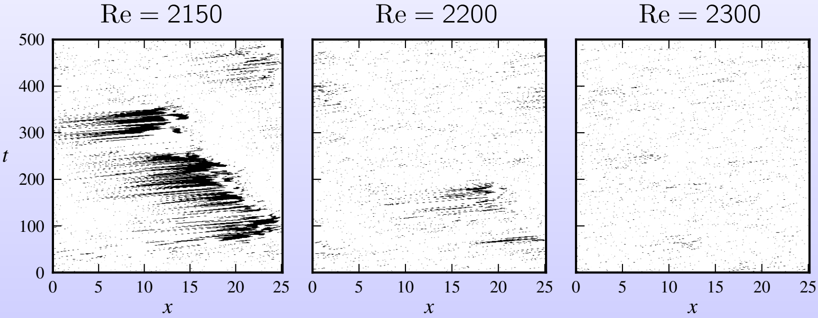
Given a small positive threshold q_*, we define the *intermittency function* as

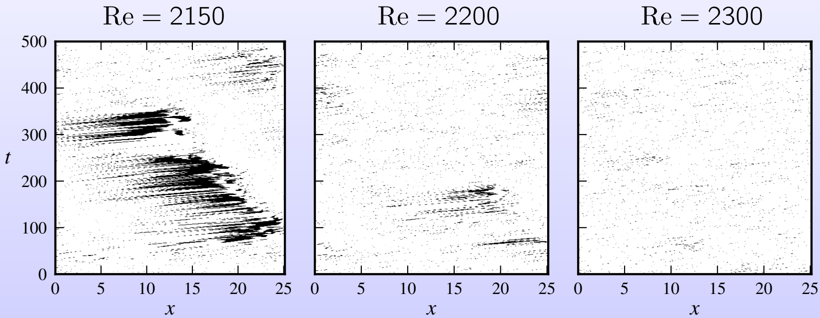
$$I(x,t) = \begin{cases} 1, & q(x,t) > q_*, \\ 0, & \text{otherwise.} \end{cases}$$

The the *intermittency factor* $0 \leq \gamma \leq 1$ is given by

$$\gamma = \langle I \rangle$$

where the average is taken over space and time.


Re = 2150
Re = 2200
Re = 2300
t
x
0
100
200
300
400
500
0
5
10
15
20
25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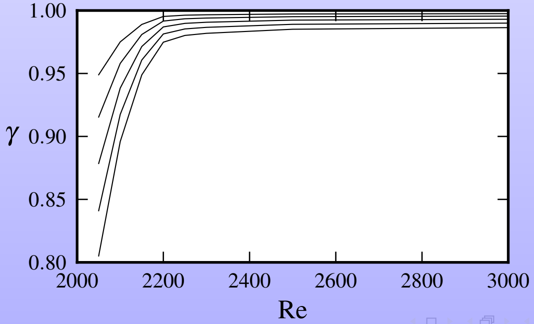
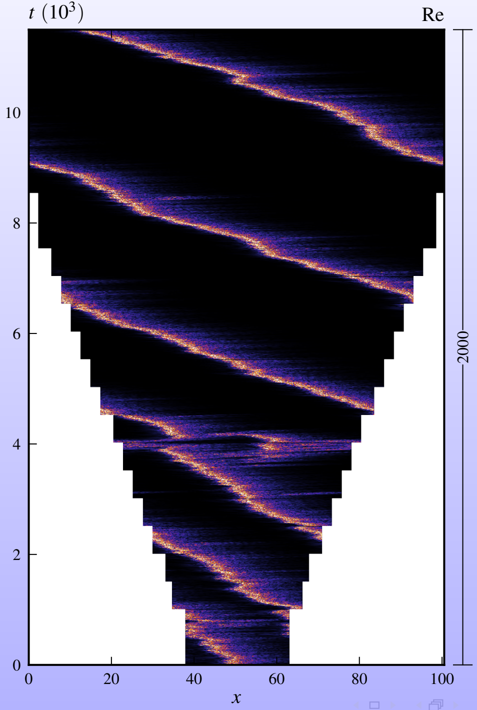

γ
Re
0.80
0.85
0.90
0.95
1.00
2000
2200
2400
2600
2800
3000

Domain Expansion

- Space-time plots show that puffs split apart often in the $\mathrm{Re} = 2050$ case, but very little at $\mathrm{Re} = 2000$. Is this natural behaviour of Navier-Stokes?
- To answer the question, we consider a puff placed in a pipe of length $L = 25D$.
- Then we expand the domain every 500 time units by $L = 5D$ until we reach $L = 100D$.
- N_x is increased with L so that the domain remains correctly resolved.
- This was done for two separate simulations at both $\mathrm{Re} = 2000$ and $\mathrm{Re} = 2050$ using the same initial condition.
- Again we plot the history data with the quantity q.



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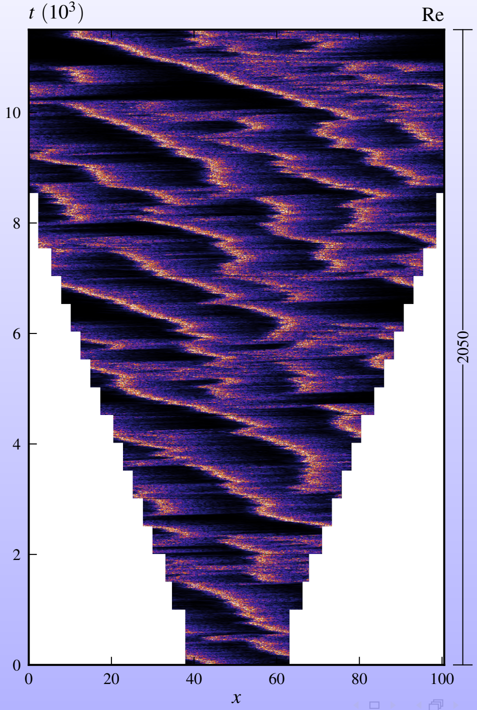

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From $\mathrm{Re} = 2000$ to 2050

- Take the single puff in the pipe at $\mathrm{Re} = 2000$ when $L = 100D$. Does increasing Re to 2050 cause the same splitting phenomenon seen earlier?

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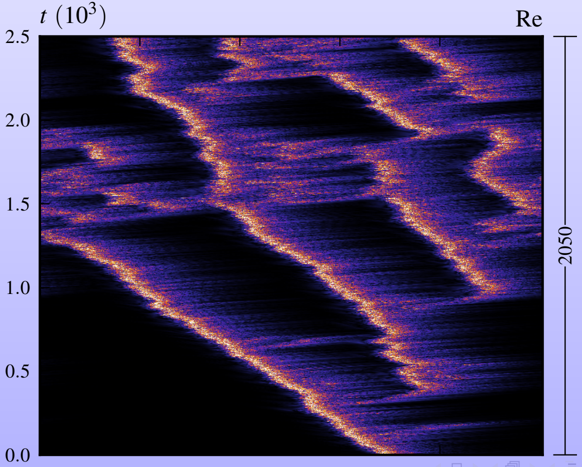

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- When Re is reduced to 2000, dynamics become **intensive**; the amount of puffs seen will depend on the initial conditions.
- **Conclusion:** for $2000 \leq \mathrm{Re} \leq 2050$ there is a critical point Re_1 after which intensive dynamics turn to extensive.

Conclusions

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  - Mechanisms for transition at Re_1 and Re_2.