A computational investigation of pipe flow at transitional Reynolds numbers

Studying the transition from turbulence

David Moxey[1], Dwight Barkley

Mathematics Institute
University of Warwick

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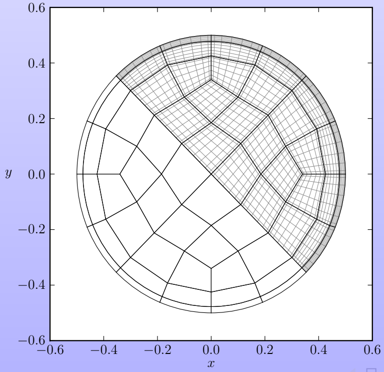
[1] D.C.Moxey@warwick.ac.uk

Introduction

- Instability in pipe flow has been an outstanding problem in fluid dynamics over the past century – first observed by Reynolds in his paper of 1883, where he defined the Reynolds number, Re.
- Generally study of the transition problem has mainly concentrated on transition from **laminar to turbulent** flow. We wish to build a phase portrait of the fluid going from **turbulent to laminar** flow, and investigate states of **laminar-turbulent co-existance**.
- This talk outlines results that we have obtained from direct numerical simulations (DNS) of the pipe flow problem at transitional Reynolds numbers.
- All of the numerical results presented here used the computing resources at the Centre for Scientific Computing, University of Warwick.

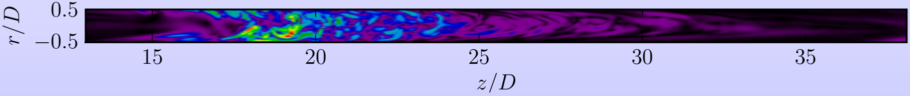
Semtex

- Our simulations use an existing DNS code, `Semtex` [Blackburn & Sherwin, *J. Comput. Phys.*, 2004].
  - 2D Spectral-element discretization in circular cross-sections.
  - Fourier pseudo-spectral down length of pipe (periodic boundary conditions).
  - Parallelized using MPI.



Large-scale structures

- For $1900 \leq \mathrm{Re} \leq 2200$, we observe *puffs* in the fluid. These turbulent structures co-exist with laminar flow, and may be important in understanding transitional behaviour.



- This puff was recorded at $\mathrm{Re} = 2000$ and has length $L \approx 25D$, so larger pipe lengths are needed to capture their behaviour.

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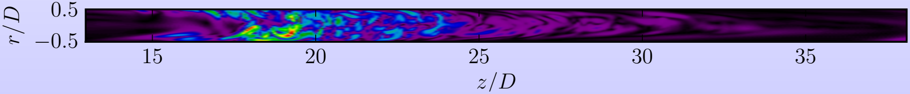


- This puff was recorded at $\mathrm{Re} = 2000$ and has length $L \approx 25D$, so larger pipe lengths are needed to capture their behaviour.
- Most significant question for us: do puffs arise in the natural transition from turbulent to laminar flow? If so, how do they form, and does Reynolds number affect their behaviour?

Simulations

- Parameters for the simulations were:

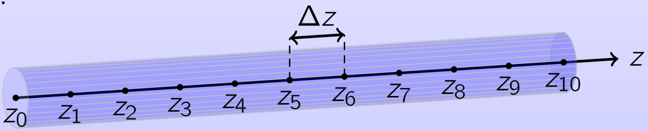
L	$16\pi D \approx 50D$	$48\pi D \approx 150D$
N_z	768	2048
N_{proc}	64	128
Re	$1900 \leq \text{Re} \leq 2150$	$2000 \leq \text{Re} \leq 2500$
Δt	2×10^{-3}	2×10^{-3}

- The flow is driven using a constant volumetric flux $q = 1 \Rightarrow U_B$ is fixed.
- The simulation is started with uniform turbulence using a short run at $\text{Re} = 5000$. We then reduce Re as follows:

$$
\begin{array}{ll}
5000 \to 4000 \to 3000 & \text{500 time units} \\
2800 \to 2650 \to 2500 \to 2350 & \text{1000 time units} \\
2200 \to 2150 \to 2100 \to 2050 \to \cdots & \text{2000 time units}
\end{array}
$$

History Plots

- Field data is too large to record regularly. We measure *history data*: every 0.1 time units, we record **v** and p from points along the axis of the pipe.



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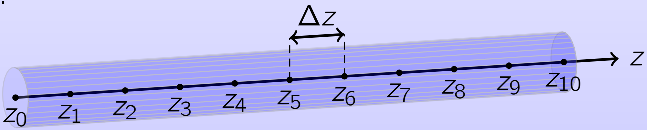


- For the velocity field $\mathbf{u} = (u, v, w)$, we construct the quantity

$$q(z,t) = \sqrt{u^2 + v^2}\Big|_{(x=0,y=0,z,t)} = \sqrt{2E_{\text{transverse}}}$$

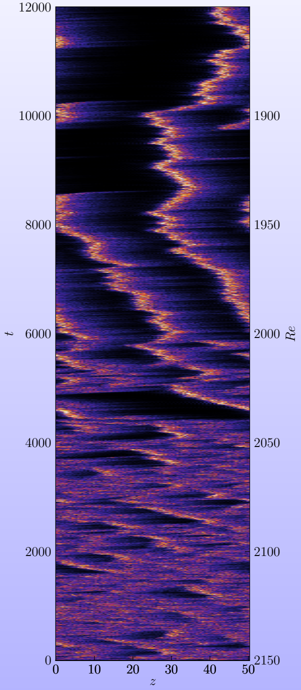
and then change to a moving frame of reference by applying

$$q(z,t) \to q(z - ct, t)$$

where, generally, $c = U_B$.

$L = 50D$ History Data

- $1900 \leq \mathrm{Re} \leq 2150$.
- $0 \leq t \leq 12000$.
- $c = U_B$.
- Re-laminarization occurs at $\mathrm{Re} = 1750$.
- The data took roughly 4.1 CPU years to generate.



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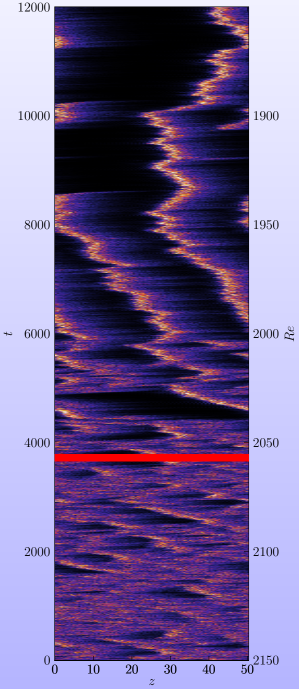
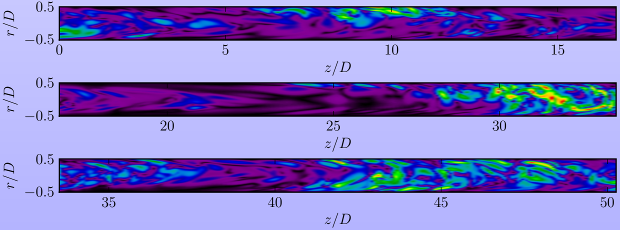


Re = 2100, $t = 3900$



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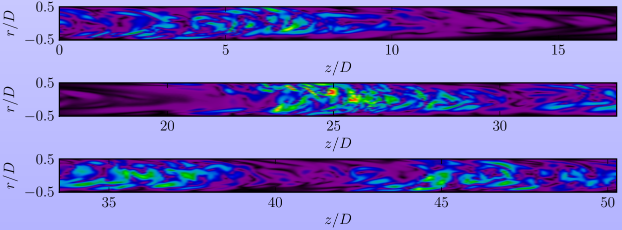
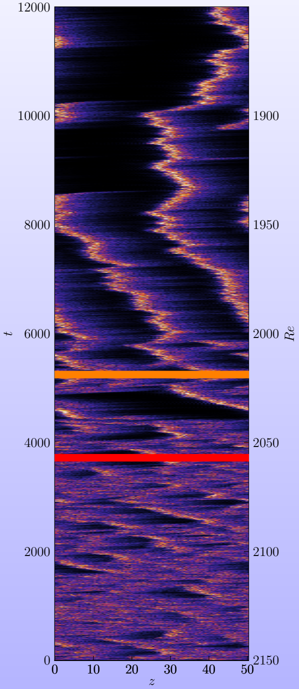
Re $= 2050$, $t = 5400$





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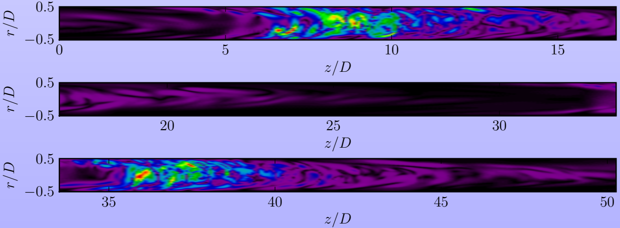
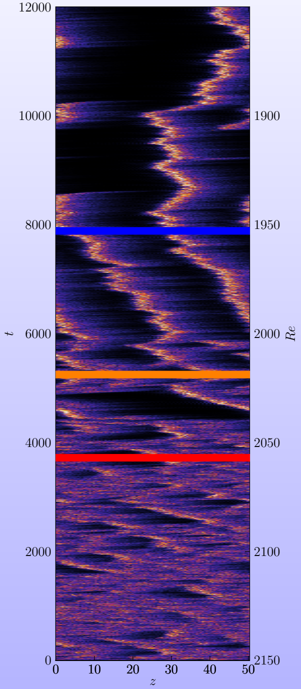
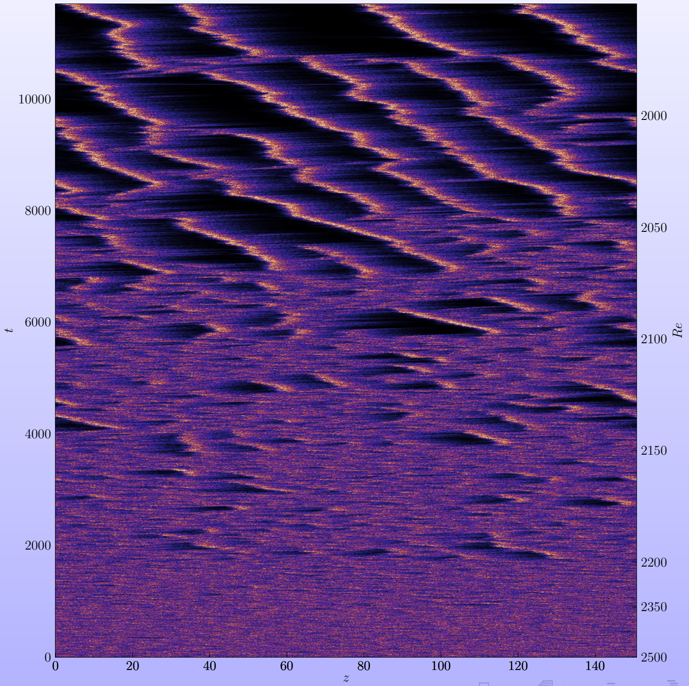
Re $= 2000$, $t = 7950$







10000
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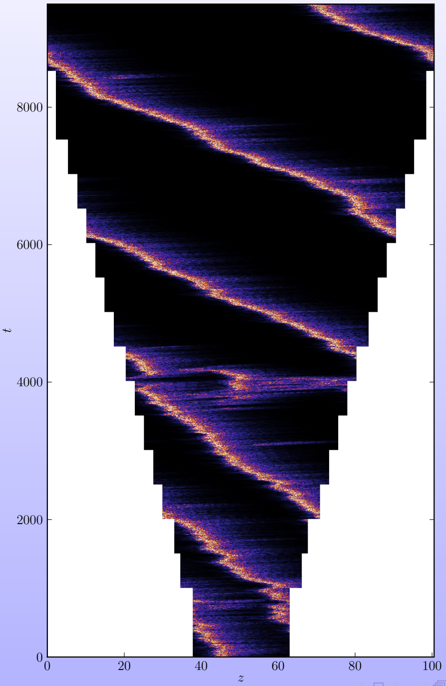
120
140
z
2000
2050
2100
2150
2200
2350
2500
Re

Domain Expansion

- We saw that puffs tended to split apart in the Re $= 2050$ case. Is this just a co-incidence or more of a natural behaviour of Navier-Stokes?
- To answer the question, we take a puff of $L = 25D$.
- Then we expand the domain every 500 time units by $L = 5D$ until we reach $L = 100D$.
- N_z is increased with L so that the domain remains correctly resolved.
- This was done for two separate simulations at both Re $= 2000$ and Re $= 2050$.
- Again we plot the history data with the quantity q.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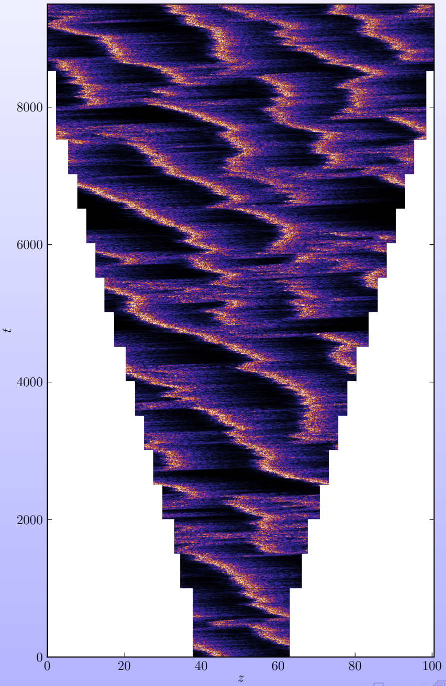
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Conclusions

- Space-time plots reveal spontaneous appearance of puff states in transition from turbulence; also see natural appearance of trains of puffs.
- Domain expansion of the puffs reveals that puffs tend to split at $\mathrm{Re} = 2050$.
- We believe that the more ‘intermittant’ behaviour found in the latter case is worth investigating in far greater detail.
- Future work: development of a method for quantitatively identifying onset of intermittency.